Gauge Invariant Renormalizability of Quantum Gravity

量子引力的规范不变可重整性

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Contents

目录

References 421

参考文献 421

Abstract

摘要

The current understanding of renormalization in quantum gravity (QG) is based on the fact that UV divergences of effective actions in the covariant QG models are covariant local expressions. This fundamental statement plays a central role in QG, and, therefore, it is important to prove it for the widest possible range of the QG theories. Using the Batalin-Vilkovisky technique and the background field method, we elaborate the proof of gauge invariant renormalizability for a generic model of quantum gravity that is diffeomorphism invariant and does not have additional, potentially anomalous, symmetries.

当前量子引力 (QG) 对重整化的理解基于以下事实: 协变量子引力模型中有效作用量的紫外发散是协变定域表达式。这一基础命题在量子引力中占据核心地位，因此，在尽可能广泛的量子引力理论中证明该命题十分重要。我们利用巴塔林-维尔可夫斯基技术与背景场方法，针对任意微分同胚不变、且不带有额外潜在反常对称性的量子引力通用模型，完成了规范不变可重整性的证明。

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关键词

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量子引力 · BRST · 巴塔林-维尔可维斯基方法 · 背景场方法 - 可重整性

Introduction

引言

All the fundamental interactions that exist in nature (electroweak, strong, and gravitational) are described in terms of gauge theories [1]. Thus, the quantization of gauge theories that provides important insights into the quantum properties of the fundamental forces plays an important role in this process. General relativity (GR) and most of its extensions are based on the general covariance principle. In the models where this symmetry does not hold at the classical level (let us note that there is a special chapter in this section, devoted to the Hořava, or Hořava-Lifshitz, gravity models, in which the covariance is violated), this symmetry is supposed to restore in the limit corresponding to observations. Thus, the basic models of QG have to be based on the covariant classical theory.

自然界中所有存在的基本相互作用 (电弱相互作用、强相互作用与引力相互作用) 都可以用规范理论描述 [1]。因此，能够为基本力的量子性质提供重要洞察的规范理论量子化，在这一研究中发挥着重要作用。广义相对论 (GR) 及其大多数扩展理论都建立在广义协变原理的基础上。对于那些该对称性在经典层面不成立的模型 (请注意，本节有一个专门章节介绍霍拉瓦引力，即霍拉瓦-利夫希茨引力模型，这类模型中协变性不成立)，该对称性应当会在对应观测的极限下恢复。因此，量子引力 (QG) 的基础模型必须建立在协变经典理论之上。

At the classical level, the coordinate transformations change the metric (or other fields, describing gravity, for example, independent connection, or tetrad, etc.) to the equivalent one, from a physical viewpoint. The situation is the same as in other gauge theories, with the coordinate invariance playing the role of gauge symmetry. In this review, we discuss only the QG models in which gravity is described by the metric. Thus, we are leaving aside such interesting issues as first-order (Palatini) formalism in QG [2], teleparallel gravity [3], etc. Let us note, by passing, that the quantum aspects of the teleparallel gravity are not well elaborated.

在经典层面，坐标变换会将度规 (或其他描述引力的场，例如独立联络、标架等等) 变换为物理视角下等价的度规。这一情况和其他规范理论相同，其中坐标不变性发挥着规范对称性的作用。本综述仅讨论引力由度规描述的量子引力模型，因此我们没有涵盖一阶 (帕拉蒂尼) 量子引力形式体系 [2]、远平行引力 [3] 等有趣课题。顺便一提，远平行引力的量子方面目前尚未得到充分研究。

In quantum theory, the gauge symmetry results in degeneracy and difficulties in the immediate application of quantum mechanics, in both Hamiltonian (canonical) and Lagrangian (covariant) formalisms. In this section, we concentrate our attention on the Lagrangian approach and the path integral technique, which is an essential ingredient of covariant quantization and provides economic way of getting the Feynman rules directly from the classical Lagrangian.

在量子理论中，规范对称性会导致简并，使得量子力学无论在哈密顿 (正则) 形式体系还是拉格朗日 (协变) 形式体系中都无法直接应用，存在诸多困难。本节我们将重点关注拉格朗日方法与路径积分技术，后者是协变量子化的核心组成部分，它提供了一种直接从经典拉格朗日量得到费曼规则的简便方法。

The relevant question is what happens with the diffeomorphism invariance at the quantum level. In particular, is it true that this symmetry holds after the loop corrections are taken into account? Or, in a more particular way, does the general covariance hold in the counterterms that cancel the divergences of the effective action in the model of QG? In what follows, we shall see that the answer to both questions may be positive for a variety of the QG models.

一个关键问题是，微分同胚不变性在量子层面会发生什么。具体来说，在考虑圈修正后，该对称性仍然成立吗？更具体地问，在抵消量子引力模型中有效作用量发散的抵消项里，广义协变性仍然成立吗？在下文中我们将看到，对于多种量子引力模型而言，两个问题的答案都可能是肯定的。

Since QG represents a particular type of gauge theories, the quantization of gravity should follow the same rules as in other gauge theories. The covariant quantization of gauge theories has made a long way starting from the famous work of Feynman [4] where the non-unitarity of S -matrix in Yang-Mills [5] and gravity theories has been found within the approach based on a naive quantization procedure. This procedure was developed in quantum electrodynamics (QED), which is an Abelian gauge theory. Later on, consistent covariant quantization of the Yang-Mills theories has been found in the papers by Faddeev and Popov [6], and DeWitt [7]. The functional integration measure that appears in the course of quantization [7] can be encoded into an additional functional integration over the so-called ghost fields or Faddeev-Popov ghosts [6]. The Faddeev-Popov method operates with the complete (also called total) quantum action which is a sum of the initial classical action, the ghost action, and the gauge-fixing action. The total action should be used for the construction of generating functional of Green functions.

由于量子引力 (QG) 属于一类特殊的规范理论，引力的量子化应当遵循与其他规范理论相同的规则。规范理论的协变量子化发展历程悠久，始于费曼的著名研究 [4]，该研究发现在基于朴素量子化程序的方法中，杨-米尔斯理论 [5] 和引力理论中的 S 矩阵不具备幺正性。这套朴素程序最早是在量子电动力学 (QED，一种阿贝尔规范理论) 中发展出来的。随后，法捷耶夫、波波夫 [6] 和德维特 [7] 在各自研究中找到了杨-米尔斯理论自洽的协变量子化方法。量子化过程 [7] 中出现的泛函积分测度可以通过对所谓鬼场 (即法捷耶夫-波波夫鬼场 [6]) 的额外泛函积分来表述。法捷耶夫-波波夫方法基于完全 (也称为总) 量子作用量运作，总量子作用量是初始经典作用量、鬼作用量与规范固定作用量之和，构造格林函数的生成泛函必须使用该总作用量。

The first proof of the gauge invariant renormalizability in the Yang-Mills theories has been given by 't Hooft and Veltman in [8, 9], which was an important contribution to creating the quantum field theory (QFT) foundations of the standard model of particle physics and its extensions. These works have been awarded the 1999 Nobel Prize in Physics "for elucidating the quantum structure of electroweak interactions in physics." The proof of renormalizability was based on the basic Ward identities, which was a very complicated approach. Soon after that, there were developed new powerful techniques of exploring non-Abelian gauge symmetries at the quantum level. One of the most remarkable discoveries in this respect was that the total Faddeev-Popov-DeWitt action is invariant under global supersymmetry called the BRST symmetry (named by Becchi, Rouet, Stora, and Tyutin) [10-12]). The nilpotent BRST charge plays a crucial role in the construction of the physical state space of any Yang-Mills -type theory allowing to define the gauge-independent and unitary S -matrix [13].

杨-米尔斯理论中规范不变可重整性的首个证明由特霍夫特和韦尔特曼在文献 [8, 9] 中给出，这为粒子物理标准模型及其扩展的量子场论 (QFT) 基础构建做出了重要贡献。这些研究成果也为他们赢得了 1999 年诺贝尔物理学奖，获奖理由是“阐明了物理学中电弱相互作用的量子结构”。该可重整性证明基于基本沃德恒等式，是一种极为复杂的研究方法。不久之后，学界发展出了更强大的新技术，用于探索量子层面的非阿贝尔规范对称性。这一领域最值得关注的发现之一是: 法捷耶夫-波波夫-德维特总作用量在一种全局超对称性下保持不变，该对称性被称为 BRST 对称性 (以贝基、鲁埃、斯托拉和秋田的名字命名)[10-12]。幂零 BRST 荷在构建任意杨-米尔斯型理论的物理态空间中发挥着关键作用，它可以用来定义不依赖于规范的幺正 S 矩阵 [13]。

The most efficient way to explore the gauge symmetry at the quantum level is the Batalin-Vilkovisky technique, which is a natural extension of the DeWitt-Faddeev-Popov [6, 7] procedure and the Becchi-Rouet-Stora and Tyutin (BRST) symmetry [11, 12]. The Batalin-Vilkovisky approach can be most successfully applied in the theory of the Yang-Mills type when the generators of gauge transformations form a closed algebra. We shall see, in the next sections, that QG, with the diffeomorphism invariance as a gauge symmetry, belongs to this class of theories. Another relevant restriction is that there should not be other symmetries that can ”compete” with the diffeomorphism invariance at the quantum level. A simple criterium to see whether an additional symmetry is really spoiling the renormalizability is to check whether there is a regularization preserving both symmetries at the same time. In all known cases, the lack of such symmetry means there are quantum anomalies. A typical example is the local conformal symmetry or the Weyl symmetry. It is expected that the anomaly violates the renormalizability of the theories of QG (contrary to the QFT in curved space-time) with this symmetry at higher loop orders. However, since this is an advanced (and not very well-elaborated) subject, we shall not be concerned about it in the present review. Thus, in what follows, we assume the use of dimensional regularization and the absence of conformal symmetry.

在量子层面探索规范对称性最高效的方法是巴塔林-维尔可夫斯基 (Batalin-Vilkovisky) 技术，它是德维特-法捷耶夫-波波夫 (DeWitt-Faddeev-Popov)[6, 7] 方法以及贝基-鲁埃-斯托拉-秋金 (BRST) 对称性 [11, 12] 的自然推广。当规范变换生成元构成闭合代数时，巴塔林-维尔可夫斯基方法可以最成功地应用于杨-米尔斯型理论。我们将在后续章节看到，以微分同胚不变性作为规范对称性的量子引力 (QG) 属于这类理论。另一项相关限制是，不应存在其他能在量子层面与微分同胚不变性“竞争”的对称性。判断额外对称性是否真的会破坏可重整性的一个简单判据是，检查是否存在能同时保留两种对称性的正则化方案。在所有已知情况中，缺少这种对称性就意味着存在量子反常。典型例子是局部共形对称性，即外尔 (Weyl) 对称性。一般认为，在高阶圈图下，该反常会破坏带有这种对称性的量子引力理论的可重整性，这与弯曲时空的量子场论不同。不过，由于这是一个较为高深且尚未得到充分研究的课题，我们在本篇综述中不对此展开讨论。因此，在下文中我们假设采用维数正则化，且不存在共形对称性。

The Batalin-Vilkovisky technique implies introducing additional objects called antifields. With antifields, the BRST symmetry and the corresponding equations for the effective action (Zinn-Justin equation [14] and Slavnov-Taylor identities in Yang-Mills theory [8, 15, 16]) become more straightforward to analyze. The gauge invariant renormalizability guarantees that in all orders of loop expansion for the quantum effective action, one can control deformations of the generators of gauge transformations, which leave such an action invariant. In the background field method, one can maintain general covariance of the divergent part of effective action when the mean quantum fields, ghosts, and antifields are switched off.

巴塔林-维尔可夫斯基技术需要引入称为反场的额外对象。借助反场，BRST 对称性以及有效作用量对应的对应方程 (杨-米尔斯理论中的齐恩-贾斯汀方程 [14] 和斯拉沃诺夫-泰勒恒等式 [8, 15, 16]) 会变得更易于分析。规范不变可重整性保证，在量子有效作用量圈展开的所有阶中，我们都可以控制规范变换生成元的形变，而这类形变会保持作用量不变。在背景场方法中，当平均量子场、鬼场和反场都关闭时，我们可以保持有效作用量发散部分的广义协变性。

The traditional (and correct) view of the difficulty in quantizing the gravitational field is that the quantum GR is not a renormalizable theory, while the renormalizable version of the theory includes fourth derivatives in the action [17], and therefore it is not unitary. In the last decades, this simple two-side story was getting more complicated, with the new models of superrenormalizable gravity, both polynomial [18] and non-polynomial [19-21]. Typically, these models intend to resolve the conflict between non-renormalizability and non-unitarity by introducing the finite or infinite amounts of derivatives compared to the fourth-derivative model [17].

关于引力场量子化困难, 传统 (且正确) 的观点是: 广义相对论的量子理论不是可重整理论, 而该理论的可重整版本在作用量中包含四阶导数，因此它不满足幺正性。近几十年来，这个简单的二元对立结论变得愈发复杂，出现了新的超可重整引力模型，包括多项式模型 [18] 和非多项式模型 [19-21]。这类模型通常通过引入比四阶导数模型 [17] 更多的有限阶或无穷阶导数，来解决不可重整与非幺正性之间的矛盾。

The main advantage of the non-polynomial models is that the tree level propagator may have the unique physical pole corresponding to the massless graviton. At the same time, the dressed propagator has, typically, an infinite (countable) amount of the ghost-like states with complex poles [22], and hence the questions about physical contents and quantum consistency of such a theory remains open, especially taking into account the problems with reflection positivity [23,24].

非多项式模型的主要优势在于, 其树级传播子可以存在对应于无质量引力子的唯一物理极点。同时, 重整化后的传播子通常存在无穷多 (可数) 带复极点的类鬼态 [22]，因此这类理论的物理内容和量子一致性问题仍悬而未决，尤其是考虑到反射正性相关问题 [23,24] 之后更是如此。

On the other hand, within the polynomial model, one can prove the unitarity of the S -matrix within the Lee-Wick approach [25] to quantum gravity in four [26] and even higher-dimensional space-times [27]. Furthermore, it is possible to make explicit one-loop calculations [28] which provide exact beta-functions in these theories due to the superrenormalizability of the theory. In the part of stability, the existing investigations concerned special backgrounds, namely, cosmological [29, 30] and black hole cases [31-33]. While the black hole results do not look conclusive, the results for the cosmological backgrounds provide a good intuitive understanding of the problem of stability in the gravity models with higher derivative ghosts.

另一方面，在多项式模型框架内，可以证明在四维 [26] 乃至更高维时空 [27] 的量子引力中，利用李-威克 (Lee-Wick) 方法 [25] 得到的 S 矩阵满足幺正性。此外，由于理论具有超可重整性，可以进行显式单圈计算 [28]，得到这些理论中精确的 β 函数。在稳定性方面，现有研究关注了特殊背景，即宇宙学背景 [29, 30] 和黑洞背景 [31-33]。黑洞背景的相关结果尚无定论，而宇宙学背景的结果为我们提供了对带高阶导数鬼的引力模型稳定性问题的良好直观理解。

Independent on the efforts in better understanding the role of ghosts and instabilities in both polynomial and non-polynomial models, it would be useful to have formal proof that, at the quantum level, these theories are renormalizable or superrenormalizable. The first proof was given for the fourth derivative quantum gravity [17], with some simplifications and generalizations achieved in [34,35] as an application of a general approach [36]. Recently, there was a renewed interest in this subject, in particular, the proofs of the general-covariant renormalizability in the general types of local [37] models of QG and even a more general Batalin-Vilkovisky-based proof of a gauge invariant renormalizability in the general models of quantum gravity, which may include higher derivatives, including polynomial or non-polynomial models [38]. In what follows, we shall closely follow the last reference, just adding a few more details.

> 无论在更好理解虚粒子和不稳定性对多项式与非多项式模型的作用方面取得了多少进展，若能给出形式化证明证明这些理论在量子层面是可重整或超可重整的，将会大有裨益。四阶导数量子引力的首个证明已在文献 [17] 中给出，文献 [34,35] 作为通用方法 [36] 的应用，完成了部分简化与推广。近来该领域重新引发关注，尤其是针对各类局域 [37] 量子引力模型完成了广义协变可重整性证明，甚至出现了基于巴达林-维尔科夫斯基方法、适用于更广泛量子引力通用模型的规范不变可重整性证明，这类模型可包含高阶导数，也涵盖多项式与非多项式模型 [38]。下文我们将紧密遵循最后这篇文献的内容，仅补充若干细节。

The review is organized as follows: In section ”Quantum Gravity in the Background Field Formalism,” we formulate the Batalin-Vilkovisky formalism combined with the background field method in QG. In section ”Gauge Invariant Renor-malizability,” this formalism is applied to the formal proof of renormalizability in the model of QG of the general form. On top of that, we discuss the gauge fixing independence of the vacuum functional and of the gravitational S -matrix.

> 本综述结构安排如下: 在“背景场形式体系下的量子引力”小节中，我们构建了结合巴达林-维尔科夫斯基形式体系与背景场方法的量子引力框架。在“规范不变可重整性”小节中，我们将该形式体系应用于通用形式量子引力模型可重整性的形式化证明。此外，我们还讨论了真空泛函与引力 S 矩阵的规范固定无关性。

Section ”A Short Historical Review and More Special Notes” contains the short historic note, the extended list of the main references (which certainly remains incomplete owing to the limited size of the review), and a brief discussion of the requirements on the gauge theory which enable one to use the Batalin-Vilkovisky formalism. In section ”On the Gauge Fixing in the Higher Derivative Models,” the gauge fixing in the higher derivative models is discussed. Finally, in section ”Conclusions,” we draw our conclusions.

> “简史回顾与补充说明”小节包含简要历史梳理、主要参考文献的扩展列表 (受限于综述篇幅，该列表肯定仍不完整)，以及对可应用巴达林-维尔科夫斯基形式体系的规范理论所需满足条件的简要讨论。在“高阶导数模型的规范固定”小节中，我们讨论了高阶导数模型的规范固定问题。最后在“结论”小节中给出本文结论。

Condensed DeWitt’s notations [39] are used in the review. Right and left derivatives of a quantity f with respect to the variable φ are denoted as $\frac{\delta_r f}{\delta\varphi}$ and $\frac{\delta_l f}{\delta\varphi}$, correspondingly. The Grassmann parity and the ghost number of a quantity A are denoted by $\varepsilon(A)$ and $\mathrm{gh}(A)$, respectively. The reader can see Eq. (22) for the definition, in the last case. For the sake of generality, we perform all general considerations for an arbitrary space-time dimension D . Let us note that this is different from the previous chapter of this section of the handbook, where it was assumed $D = 4$. The condensed notation for the space-time integral in D dimensions, $\int dx = \int d^D x$, is used throughout the text of the present chapter.

本综述采用狄维特简记法 [39]。量 f 对变量 φ 的右导数和左导数分别记为 $\frac{\delta_r f}{\delta\varphi}$ 和 $\frac{\delta_l f}{\delta\varphi}$ 。量 A 的格拉斯曼奇偶性和鬼数分别记为 $\varepsilon(A)$ 和 $\mathrm{gh}(A)$，读者可参见式 (22) 获取后者的定义。为保证通用性，我们针对任意时空维数 D 开展所有一般性讨论。请注意这与本手册本节前一章的假设不同，前一章默认满足 $D=4$ 。本章全文统一使用 D 维时空积分的简记记法 $\int dx = \int d^D x$ 。

Quantum Gravity in the Background Field Formalism

背景场形式论中的量子引力

Our purpose is to explore the gravitational theory based on an arbitrary action of a Riemann’s metric $S_0(g)$, where $g = g_{\mu\nu}(x)$. In what follows, we usually omit the indices in the arguments of functions or functionals. The action is assumed invariant under the general coordinate transformations:

我们的目的是探索基于任意黎曼度量作用量 $S_0(g)$ 的引力理论，其中 $g = g_{\mu\nu}(x)$。在下文中，我们通常会省略函数或泛函自变量中的指标。假设该作用量在一般坐标变换下保持不变:

$$x'^{\mu} = x'^{\mu}(x),\ g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x). \tag{1}$$

The standard examples of the theories of our interest are Einstein’s gravity (GR) with a cosmological constant term:

我们所关注的理论的标准例子是带宇宙常数项的爱因斯坦引力 (广义相对论):

$$S_{EH}(g) = -\frac{1}{\kappa^2}\int dx\sqrt{-g}\,(R + 2\Lambda) \tag{2}$$

and a general version of higher derivative gravity. The corresponding action is described in Chap. 8, ”The Background Information About Perturbative Quantum Gravity”:

以及高阶导数引力的一般形式。对应的作用量已在第 8 章“微扰量子引力背景知识”中介绍:

$$S(g) = S_{EH}(g) + \int dx\sqrt{-g}\Big\{R^{\mu\nu\alpha\beta}\Pi_1\Big(\frac{\Box}{M^2}\Big)R_{\mu\nu\alpha\beta}$$

$$+R^{\mu\nu}\Pi_2\Big(\frac{\Box}{M^2}\Big)R_{\mu\nu} + R\Pi_3\Big(\frac{\Box}{M^2}\Big)R + \mathcal{O}\big(R^3_{\dots}\big)\Big\}. \tag{3}$$

Here, $\Pi_{1,2,3}$ are polynomial or non-polynomial form factors in the quadratic in curvatures part of the Lagrangian, and the last term represents non-quadratic in curvature terms. In quantum theory, action (3) may lead to the theory which is non-renormalizable, renormalizable, or superrenormalizable, depending on the choice of the functions $\Pi_{1,2,3}(x)$ and the non-quadratic terms.

此处，$\Pi_{1,2,3}$ 是拉格朗日中曲率二次项部分的多项式或非多项式形状因子，最后一项代表曲率中的非二次项。在量子理论中，取决于函数 $\Pi_{1,2,3}(x)$ 和非二次项的选择，作用量 (3) 导出的理论可以是非重整化、可重整化或超可重整化的。

The dimensional parameter M^2 in the form factors $\Pi_{1,2,3}$ is a universal mass scale at which the QG effect becomes relevant. For instance, M^2 can be the square of the Planck mass, but there may be other options, including multiple scale models, as analyzed in [40]. For the analysis presented below, these (otherwise important) details are irrelevant since the unique necessary feature is that the action $S(g)$ is diffeomorphism invariant.

形状因子 $\Pi_{1,2,3}$ 中的量纲参数 M^2 是量子引力效应开始显现的普适质量标度。例如，M^2 可以是普朗克质量的平方，但也存在其他可能，包括文献 [40] 中分析的多标度模型。对于下文的分析，这些 (尽管本身很重要的) 细节并不相关，因为唯一必要的条件是作用量 $S(g)$ 满足微分同胚不变性。

In the infinitesimal form, the transformations (1) read

变换 (1) 的无穷小形式可以写为

$$x'^{\mu} = x^{\mu} + \xi^{\mu}(x) \Rightarrow g'_{\mu\upsilon}(x) = g_{\mu\upsilon}(x) + \delta g_{\mu\upsilon}(x), \tag{4}$$

where

其中

$$\delta g_{\mu\nu}(x) = -\xi^{\sigma}(x)\,\partial_{\sigma} g_{\mu\nu}(x) - g_{\mu\sigma}(x)\,\partial_{\nu}\xi^{\sigma}(x) - g_{\sigma\nu}(x)\,\partial_{\mu}\xi^{\sigma}(x). \tag{5}$$

On top of (5), we also need the transformation rule for vector fields $A_{\mu}(x)$ and $A^{\mu}(x)$,

除了 (5) 式，我们还需要矢量场 $A_{\mu}(x)$ 和 $A^{\mu}(x)$ 的变换规则,

$$\delta A_{\mu}(x) = -\xi^{\sigma}(x)\,\partial_{\sigma} A_{\mu}(x) - A_{\sigma}(x)\,\partial_{\mu}\xi^{\sigma}(x), \tag{6}$$

$$\delta A^{\mu}(x) = -\xi^{\sigma}(x)\,\partial_{\sigma} A^{\mu}(x) + A^{\sigma}(x)\,\partial_{\sigma}\xi^{\mu}(x). \tag{7}$$

The invariance of the action $S_0(g)$ under the transformations (5) can be expressed in the form of Noether identity:

作用量 $S_0(g)$ 在变换 (5) 下的不变性可以用诺特恒等式表示为:

$$\int dx \frac{\delta S_0(g)}{\delta g_{\mu\nu}(x)} \delta g_{\mu\nu}(x) = 0. \tag{8}$$

Let us present the transformations (5) in the form

我们将变换 (5) 写成如下形式

$$\delta g_{\mu\nu}(x) = \int dy R_{\mu\nu\sigma}(x, y; g)\,\xi^{\sigma}(y), \tag{9}$$

where the generators of gauge transformations of the metric tensor $g_{\mu\nu}$ with gauge parameters $\xi^{\sigma}(x)$ are

其中带规范参数 $\xi^\sigma(x)$ 的度规张量 $g_{\mu\nu}$ 的规范变换生成元为

$$R_{\mu\upsilon\sigma}(x,y;g) = -\delta(x-y)\,\partial_\sigma g_{\mu\upsilon}(x) - g_{\mu\sigma}(x)\,\partial_\upsilon\delta(x-y)$$

$$-g_{\sigma\upsilon}(x)\,\partial_\mu\delta(x-y). \tag{10}$$

The algebra of generators (also called algebra of gauge transformations) can be easily verified to be

可以很容易验证，生成元的代数 (也称为规范变换代数) 为

$$\int du\left[\frac{\delta R_{\mu\nu\sigma}(x,y;g)}{\delta g_{\alpha\beta}(u)}R_{\alpha\beta\gamma}(u,z;g) - \frac{\delta R_{\mu\nu\gamma}(x,z;g)}{\delta g_{\alpha\beta}(u)}R_{\alpha\beta\sigma}(u,y;g)\right]$$

$$= -\int du R_{\mu\nu\lambda}(x,u;g)\,F^\lambda_{\sigma\gamma}(u,y,z), \tag{11}$$

where

其中

$$F^\lambda_{\alpha\beta}(x,y,z) = \delta(x-y)\,\delta^\lambda_\beta\frac{\partial}{\partial x^\alpha}\delta(x-z) - \delta(x-z)\,\delta^\lambda_\alpha\frac{\partial}{\partial x^\beta}\delta(x-y),$$

$$F^\lambda_{\alpha\beta}(x,y,z) = -F^\lambda_{\beta\alpha}(x,z,y) \tag{12}$$

are structure functions of the gauge algebra which do not depend on the metric $g_{\mu\upsilon}$.

是不依赖于度规 $g_{\mu\nu}$ 的规范代数结构函数。

The algebra (11) holds independently of the form of the covariant action. Thus, any covariant theory of gravity is a gauge theory with closed gauge algebra and with the structure functions independent of the fields (metric tensor, in this case). Thus, QG is a theory of the Yang-Mills type.

代数 (11) 的成立与协变作用量的形式无关。因此，任何协变引力理论都是闭规范代数的规范理论，且其结构函数不依赖于场 (在此情形下为度规张量)。因此，量子引力是一种杨-米尔斯型理论。

Starting from this point, the analysis of renormalization is pretty much standard, but it is important to introduce one more important element in our considerations. It proves useful to perform quantization of gravity not on the Minkowski spacetime with the constant metric tensor $\eta_{\mu\nu}$ but on undefined external background, represented by the metric tensor $\bar{g}_{\mu\nu}(x)$. Introducing an arbitrary background metric provides serious advantages by using the background field method, as we shall see in what follows. The standard references on the background field method in QFT are [7, 41, 42] . One can see also recent discussions of the method for the gauge theories in [43-46] and specifically for QG in [37].

由此出发，重整化的分析基本是标准流程，但我们的讨论中还需要引入一个重要元素。事实证明，对引力进行量子化时，不选取度规张量为 $\eta_{\mu\nu}$ 的恒定闵可夫斯基时空，而是选取由度规张量 $\bar{g}_{\mu\nu}(x)$ 描述的任意外背景场更为实用。下文我们将看到，引入任意背景度规后使用背景场方法会带来显著优势。量子场论中背景场方法的经典参考文献是 [7, 41, 42]，读者也可查阅近期 [43-46] 中对规范理论该方法的讨论，以及 [37] 中专门针对量子引力的讨论。

In the background field method, the metric $g_{\mu\nu}(x)$ is replaced by the sum

在背景场方法中，度规 $g_{\mu\nu}(x)$ 被替换为和式

$$g_{\mu\nu}(x) \to \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x), \quad \text{such that} \quad S_0(g) \to S_0(\bar{g}+h). \tag{13}$$

Here, $h_{\mu\nu}(x)$ is called quantum metric and is regarded as an integration variable in the functional integrals defining the generating functionals of Green functions.

此处，$h_{\mu\nu}(x)$ 是量子度规，在定义格林函数生成泛函的泛函积分中被视为积分变量。

The action $S_0(\bar{g}+h)$ is a functional of two variables $\bar{g}$ and h and has an additional symmetry corresponding to the transformations

作用量 $S_0(\bar{g}+h)$ 是 $\bar{g}$ 和 h 两个变量的泛函，且具有对应于下述变换的额外对称性

$$\delta\bar{g}_{\mu\nu} = \varepsilon_{\mu\nu} \quad \text{and} \quad \delta h_{\mu\nu} = -\varepsilon_{\mu\nu}, \tag{14}$$

with arbitrary symmetric tensor functions $\varepsilon_{\nu\mu} = \varepsilon_{\mu\nu} = \varepsilon_{\mu\nu}(x)$. In particular, this implies an ambiguity in defining the gauge transformations for $\bar{g}$ and h . To fix this arbitrariness, we require that the transformation of our interest has the right flat limit when $\bar{g}_{\mu\nu}(x)$ is replaced by $\eta_{\mu\nu}$. In this way, one can easily show that the gauge transformation of the quantum metric $h_{\mu\nu}$, in the presence of external (fixed) background $\bar{g}_{\mu\nu}$, has the form

其中 $\varepsilon_{\nu\mu} = \varepsilon_{\mu\nu} = \varepsilon_{\mu\nu}(x)$ 是任意对称张量函数。这尤其意味着，定义 $\bar{g}$ 和 h 的规范变换时存在歧义。为消除这种任意性，我们要求当 $\bar{g}_{\mu\nu}(x)$ 替换为 $\eta_{\mu\nu}$ 时，我们所讨论的变换具有正确的平坦极限。由此不难证明，存在外 (固定) 背景 $\bar{g}_{\mu\nu}$ 时，量子度规 $h_{\mu\nu}$ 的规范变换形式为

$$\delta h_{\mu\nu}(x) = \int dy R_{\mu\nu\sigma}(x, y; \bar{g}+h)\,\xi^{\sigma}, \tag{15}$$

while $\delta\bar{g}_{\mu\nu}(x) = 0$ and the action remains invariant, $\delta S_0(\bar{g}+h) = 0$.

同时 $\delta\bar{g}_{\mu\nu}(x) = 0$ 和作用量保持不变，$\delta S_0(\bar{g}+h) = 0$ 。

Because of the similarity with the Yang-Mills field, the Faddeev-Popov quantization procedure is quite standard (see, for example, the previous chapter of this section for the specific QG details), and the resulting action $S_{FP} = S_{FP}(\phi, \bar{g})$ has the form [6]

由于和杨-米尔斯场类似，法捷耶夫-波波夫量子化过程非常标准 (例如可见本节前一章对量子引力具体细节的讨论)，最终得到的作用量 $S_{FP} = S_{FP}(\phi, \bar{g})$ 形式如文献 [6] 所示

$$S_{FP} = S_0(\bar{g} + h) + S_{gh}(\phi, \bar{g}) + S_{gf}(\phi, \bar{g}), \tag{16}$$

where the contents of ϕ depends on the choice of the QG model (3), as explained below. In the presence of a background metric, the ghost action has the form

其中 ϕ 的内容取决于下文说明的量子引力模型 (3) 的选取。存在背景度规时，鬼场作用量形式为

$$S_{gh}(\phi, \bar{g}) = \int dxdydz\sqrt{-\bar{g}(x)}\bar{C}^{\alpha}(x) H_{\alpha}^{\beta\gamma}(x, y; \bar{g}, h) R_{\beta\gamma\sigma}(y, z; \bar{g} + h) C^{\sigma}(z),$$

(17)

with the notation

采用记号

$$H_{\alpha}^{\beta\gamma}(x, y; \bar{g}, h) = \frac{\delta\chi_{\alpha}(x; \bar{g}, h)}{\delta h_{\beta\gamma}(y)}. \tag{18}$$

Furthermore, $S_{gf}(\bar{g}, h)$ is the gauge-fixing action

此外，$S_{gf}(\bar{g}, h)$ 是规范固定作用量

$$S_{gf}(\phi, \bar{g}) = \int dx\sqrt{-\bar{g}(x)}B^{\alpha}(x)\chi_{\alpha}(x; \bar{g}, h). \tag{19}$$

which corresponds to the singular gauge condition. For the non-singular gauge condition, the action has the form

它对应奇异规范条件。对于非奇异规范条件，作用量形式为

$$S_{gf}(\phi, \bar{g}) = \int dx\sqrt{-\bar{g}(x)}\Big[B^{\alpha}(x)\chi_{\alpha}(x; \bar{g}, h) + \frac{1}{2}B^{\alpha}(x)\bar{g}_{\alpha\beta}(x)B^{\beta}(x)\Big]. \tag{20}$$

The reader can easily identify (20) as the particular version of the general gauge-fixing choice discussed in Chap. 8, ”The Background Information About Perturbative Quantum Gravity”, and also note that (19) is a degenerate version of (20). In what follows, we shall use the form (19), which is most useful for exploring the general features of renormalization. Finally, $\chi_{\alpha}(x; \bar{g}, h)$ are the gauge fixing functions. It is worth mentioning that the generalization to a more complicated nonsingular gauge fixing, required in the higher derivative theories, is considered in section ”On the Gauge Fixing in the Higher Derivative Models.”

读者不难发现，(20) 是第 8 章“微扰量子引力背景知识”中讨论的一般规范固定选取的特殊形式，也可注意到 (19) 是 (20) 的退化版本。下文中我们将使用形式 (19)，它对探究重整化的一般性质最有用。最后，$\chi_{\alpha}(x; \bar{g}, h)$ 是规范固定函数。值得一提的是，高阶导数理论所需的、推广到更复杂非奇异规范固定的内容，会在“高阶导数模型中的规范固定”一节中讨论。

Now, we are in a position to introduce an important notation used in (16):

现在我们可以给出 (16) 中用到的重要记号:

$$\phi = \{\phi^i\} = \{h_{\mu\nu}, B^\alpha, C^\alpha, \bar{C}^\alpha\}, \tag{21}$$

which is the full set of quantum fields including quantum metric, Faddeev-Popov ghost, anti-ghost, and the Nakanishi-Lautrup auxiliary fields B^α .

它是包含量子度规、法捷耶夫-波波夫鬼场、反鬼场以及中西-劳特鲁普辅助场 B^α 在内的全套量子场。

We will need two sets of numbers characterizing the quantum fields. The Grassmann parity of the fields ϕ^i will be denoted as $\varepsilon(\phi^i) = \varepsilon_i$. By definition, for ghost and anti-ghost, we have $\varepsilon(C^\alpha) = \varepsilon(\bar{C}^\alpha) = 1$, while for the auxiliary fields B^α and the quantum metric, we have $\varepsilon(B^\alpha) = \varepsilon(h_{\mu\nu}) = 0$.

我们需要两组参数来表征量子场。场 ϕ^i 的格拉斯曼奇偶性记为 $\varepsilon(\phi^i) = \varepsilon_i$ 。根据定义，对于鬼场和反鬼场，有 $\varepsilon(C^\alpha) = \varepsilon(\bar{C}^\alpha) = 1$ ，而对于辅助场 B^α 和量子度规，有 $\varepsilon(B^\alpha) = \varepsilon(h_{\mu\nu}) = 0$ 。

On top of this, another conserved quantity, called ghost number, can be defined for the same fields as

除此之外，我们还可以为这些场定义另一个称为鬼数的守恒量，即

$$\mathrm{gh}(C^\alpha) = 1,\ \mathrm{gh}(\bar{C}^\alpha) = -1 \ \text{ and } \ \mathrm{gh}(B^\alpha) = \mathrm{gh}(h_{\mu\nu}) = 0. \tag{22}$$

Consider the gravitational BRST transformations, which were introduced in [17, 47,48]. For any admissible choice of gauge-fixing functions $\chi_\alpha(x; \bar{g}, h)$, action (16) is invariant under global supersymmetry (BRST symmetry) [10,12]:

我们来考虑引力 BRST 变换，该变换已在文献 [17, 47,48] 中引入。对于任意可接受的规范固定函数 $\chi_\alpha(x; \bar{g}, h)$ ，作用量 (16) 在整体超对称 (BRST 对称性)[10,12] 下保持不变:

$$\delta_B h_{\mu\nu}(x) = \int dy R_{\mu\nu\alpha}(x, y; \bar{g} + h) C^\alpha(y) \mu,$$

$$\delta_B B^\alpha(x) = 0$$

$$\delta_B C^\alpha(x) = -C^\sigma(x) \partial_\sigma C^\alpha(x) \mu, \tag{23}$$

$$\delta_B \bar{C}^\alpha(x) = B^\alpha(x) \mu,$$

where μ is a constant Grassmann parameter, such that $\mu^2 = 0$. It is a remarkable fact that the BRST transformations (23) do not depend on gauge-fixing condition. One can present the BRST transformations (23) in the form

其中 μ 是常数格拉斯曼参数，满足 $\mu^2 = 0$。值得注意的是，BRST 变换 (23) 不依赖于规范固定条件。我们可以将 BRST 变换 (23) 写为如下形式

$$\delta_B \phi^i(x) = R^i(x; \phi, \bar{g})\mu, \tag{24}$$

where $R^i = \left\{R^{(h)}_{\mu\nu}, R^{\alpha}_{(B)}, R^{\alpha}_{(C)}, R^{\alpha}_{(\bar{C})}\right\}$ and

其中 $R^i = \left\{R^{(h)}_{\mu\nu}, R^{\alpha}_{(B)}, R^{\alpha}_{(C)}, R^{\alpha}_{(\bar{C})}\right\}$ 且

$$R^{(h)}_{\mu\nu}(x; \phi, \bar{g}) = \int dy R_{\mu\nu\sigma}(x, y; \bar{g} + h) C^{\sigma}(y),$$

$$R^{\alpha}_{(B)}(x; \phi, \bar{g}) = 0$$

$$R^{\alpha}_{(C)}(x; \phi, \bar{g}) = -C^{\sigma}(x)\partial_{\sigma}C^{\alpha}(x),$$

$$R^{\alpha}_{(\bar{C})}(x; \phi, \bar{g}) = B^{\alpha}(x). \tag{25}$$

It is easy to see that, in all cases, $\varepsilon\left(R^i(x; \phi, \bar{g})\right) = \varepsilon_i + 1$.

不难看出，在所有情况下都满足 $\varepsilon\left(R^i(x; \phi, \bar{g})\right) = \varepsilon_i + 1$。

There is an important nilpotency property of the BRST transformations (23), playing a significant role in the proof of gauge independence of the S -matrix on quantum level. To discuss this property, it is useful to introduce the BRST operator, $\hat{s}$, defined by its action of the fields

BRST 变换 (23) 具有一个重要的幂零性，它在量子层面证明 S 矩阵的规范无关性中发挥着关键作用。为讨论这一性质，引入 BRST 算符 $\hat{s}$ 是很方便的，其对场的作用定义为

$$\delta_B \phi^i = \hat{s}\phi^i \mu, \ \varepsilon(\hat{s}) = 1. \tag{26}$$

Let us formulate the proof of nilpotency using the condensed DeWitt's notations. Acting twice on all the quantum fields and using the explicit form (23), we get

我们用浓缩的德维特 notation 来表述幂零性的证明。将变换对所有量子场作用两次，并利用显式形式 (23)，我们得到

$$\hat{s}^2 B^{\alpha} = \hat{s}(\hat{s}B^{\alpha}) = \hat{s}0 = 0$$

$$\hat{s}^2 \bar{C}^{\alpha} = \hat{s}\left(\hat{s}\bar{C}^{\alpha}\right) = \hat{s}B^{\alpha} = 0,$$

$$\hat{s}^2 C^{\alpha} = \hat{s}(\hat{s}C^{\alpha}) = -\hat{s}(C^{\sigma}\partial_{\sigma}C^{\alpha}) = C^{\sigma}\partial_{\sigma}(\hat{s}C^{\alpha}) - (\hat{s}C^{\sigma})\partial_{\sigma}C^{\alpha}$$

$$= -C^{\sigma}\partial_{\sigma}\left(C^{\rho}\partial_{\rho}C^{\alpha}\right) + C^{\rho}\partial_{\rho}C^{\sigma}\partial_{\sigma}C^{\alpha}$$

$$= -C^{\sigma}\partial_{\sigma}C^{\rho}\partial_{\rho}C^{\alpha} - C^{\sigma}C^{\rho}\partial_{\sigma}\partial_{\rho}C^{\alpha} + C^{\rho}\partial_{\rho}C^{\sigma}\partial_{\sigma}C^{\alpha}$$

$$= -C^{\sigma}C^{\rho}\partial_{\sigma}\partial_{\rho}C^{\alpha} = 0,$$

$$\hat{s}^2 h_{\mu\upsilon} = \hat{s}\left(\hat{s}h_{\mu\upsilon}\right) = \hat{s}\left(R_{\mu\upsilon\sigma}C^{\sigma}\right) = \left(\hat{s}R_{\mu\upsilon\sigma}\right)C^{\sigma} + R_{\mu\upsilon\sigma}\left(\hat{s}C^{\sigma}\right)$$

$$= -\partial_{\sigma}\left(\hat{s}g_{\mu\nu}\right)C^{\sigma} - \partial_{\sigma}g_{\mu\nu}\left(\hat{s}C^{\sigma}\right) - \left(\hat{s}g_{\mu\sigma}\right)\partial_{\nu}C^{\sigma}$$

$$-g_{\mu\sigma}\partial_{\nu}\left(\hat{s}C^{\sigma}\right) - \left(\hat{s}g_{\sigma\nu}\right)\partial_{\mu}C_{\sigma} - g_{\sigma\nu}\partial_{\mu}\left(\hat{s}C^{\sigma}\right)$$

$$= \partial_{\sigma}\partial_{\rho}g_{\mu\nu}C^{\rho}C^{\sigma} + \partial_{\rho}g_{\mu\nu}\partial_{\sigma}C^{\rho}C^{\sigma} + \partial_{\rho}g_{\mu\nu}C^{\sigma}\partial_{\sigma}C^{\rho}$$

$$+\partial_{\sigma}g_{\mu\rho}\partial_{\nu}C^{\rho}C^{\sigma} + \partial_{\sigma}g_{\mu\rho}C^{\sigma}\partial_{\nu}C^{\rho} + \partial_{\sigma}g_{\rho\nu}\partial_{\mu}C^{\rho}C^{\sigma} + \partial_{\sigma}g_{\rho\nu}C^{\sigma}\partial_{\nu}C^{\rho}$$

$$+g_{\mu\rho}\partial_{\sigma}\partial_{\nu}C^{\rho}C^{\sigma} + g_{\mu\rho}C^{\sigma}\partial_{\sigma}\partial_{\nu}C^{\rho} + g_{\rho\nu}\partial_{\sigma}\partial_{\mu}C^{\rho}C^{\sigma} + g_{\rho\nu}C^{\sigma}\partial_{\sigma}\partial_{\mu}C^{\rho}$$

$$+g_{\mu\rho}\partial_{\sigma}C^{\rho}\partial_{\nu}C^{\sigma} + g_{\mu\rho}\partial_{\nu}C^{\sigma}\partial_{\sigma}C^{\rho} + g_{\rho\nu}\partial_{\sigma}\partial_{\mu}C^{\rho}C^{\sigma} + g_{\rho\nu}\partial_{\mu}C^{\sigma}\partial_{\sigma}C^{\rho}$$

$$+g_{\rho\sigma}\partial\mu C^{\rho}\partial_{\nu}C^{\sigma} + g_{\rho\sigma}\partial_{\nu}C^{\sigma}\partial_{\mu}C^{\rho} = 0, \tag{27}$$

where the anti-commutativity of ghost fields and the subsequent relations

其中鬼场的反对易性以及后续关系

$$\partial_{\rho}\partial_{\sigma}\phi C^{\sigma}C^{\rho} = 0, \tag{28}$$

valid for an arbitrary function ϕ , were used.

该关系对任意函数 ϕ 都成立。

The BRST invariance of the action S_{FP} can be expressed as

作用量 S_{FP} 的 BRST 不变性可以表示为

$$\int dx \frac{\delta_r S_{FP}}{\delta\phi^i(x)} R^i(x;\phi,\bar{g}) = 0. \tag{29}$$

A compact and useful form of the invariance property (29) is called Zinn-Justin equation [14]. To get it, one has to introduce the set of additional field variables $\phi_i^*(x)$ and extend the action. The new fields $\phi_i^*(x)$

are defined to have Grassmann parities opposite to the corresponding fields $\phi^i(x)$, namely, $\varepsilon\left(\phi_i^*\right)=\varepsilon_i+1$. The extended action $S=S(\phi,\phi^*,\bar{g})$ has the form

不变性性质 (29) 的一个简洁且有用的形式称为津恩-贾斯汀方程 [14]。为得到该方程，我们需要引入一组额外的场变量 $\phi_i^*(x)$ 并扩展作用量。新场 $\phi_i^*(x)$ 的格拉斯曼奇偶性被定义为与对应场 $\phi^i(x)$ 相反，即满足 $\varepsilon\left(\phi_i^*\right)=\varepsilon_i+1$ 。扩展后的作用量 $S=S(\phi,\phi^*,\bar{g})$ 形式为

$$S=S_{FP}+\int dx\phi_i^*(x)R^i(x;\phi,\bar{g}). \tag{30}$$

It is easy to note that the new fields $\phi_i^*(x)$ play the role of sources to the BRST generators (25). With this addition, the relation (29) gets the form of the Zinn-Justin equation for the action (30):

不难发现，新场 $\phi_i^*(x)$ 扮演着 BRST 生成元 (25) 的源的角色。加入这一项后，关系式 (29) 就变成了作用量 (30) 的津恩-贾斯汀方程:

$$\int dx\frac{\delta_r S}{\delta\phi^i(x)}\frac{\delta_l S}{\delta\phi_i^*(x)}=0. \tag{31}$$

It is worth remembering that using the left and right derivatives in the last equation is relevant due to the nontrivial Grassmann parities of the involved quantities.

需要记住，由于涉及的量具有非平凡的格拉斯曼奇偶性，在上一方程中使用左导数和右导数是必要的。

Let us introduce the terminology of the Batalin-Vilkovisky formalism [49, 50]. The sources $\phi_i^*(x)$ are called antifields. Another fundamental notion is the antibracket for two arbitrary functionals of fields and antifields, $F=F(\phi,\phi^*)$ and $G=G(\phi,\phi^*)$, which is defined as

下面我们介绍巴塔林-维尔可维斯基形式 [49, 50] 的术语。源 $\phi_i^*(x)$ 被称为反场。另一个基本概念是场和反场的两个任意泛函 $F=F(\phi,\phi^*)$ 和 $G=G(\phi,\phi^*)$ 的反括号，其定义为

$$(F,G)=\int dx\left(\frac{\delta_r F}{\delta\phi^i(x)}\frac{\delta_l G}{\delta\phi_i^*(x)}-\frac{\delta_r F}{\delta\phi_i^*(x)}\frac{\delta_l G}{\delta\phi^i(x)}\right), \tag{32}$$

The antibracket obeys the following properties:

反括号满足以下性质:

(1) Grassmann parity relations

(1) 格拉斯曼奇偶性关系

$$\varepsilon((F,G))=\varepsilon(F)+\varepsilon(G)+1=\varepsilon((G,F)); \tag{33}$$

(2) Generalized antisymmetry property

(2) 广义反对称性

$$(F, G) = -(G, F)(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}; \tag{34}$$

(3) Leibniz rule

(3) 莱布尼茨法则

$$(F, GH) = (F, G) H + (F, H) G(-1)^{\varepsilon(G)\varepsilon(H)}; \tag{35}$$

(4) Generalized Jacobi identity

(4) 广义雅可比恒等式

$$((F, G), H)(-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)} + \text{cycle}(F, G, H) \equiv 0. \tag{36}$$

In terms of the antibracket, Zinn-Justin equation (31) can be written as

借助反括号，津恩-贾斯汀方程 (31) 可以写为

$$(S, S) = 0, \tag{37}$$

which is the classical master equation of Batalin-Vilkovisky formalism [49,50]. In what follows, we shall see that this equation can be generalized to the quantum domain. This generalization is extensively used to analyze renormalization in QG.

这就是巴塔林-维尔可维斯基形式的经典主方程 [49,50]。下文我们会看到，该方程可以推广到量子领域。这一推广被广泛用于分析量子引力中的重整化。

The formulation of the quantum gauge theory starts from the generating functional of Green functions in the form of functional integral:

量子规范理论的构造从格林函数生成泛函开始，其泛函积分形式为

$$Z(J, \bar{g}) = \int d\phi \exp\left\{\frac{i}{\hbar}\left[S_{FP}(\phi, \bar{g}) + J\phi\right]\right\} = \exp\left\{\frac{i}{\hbar} W(J, \bar{g})\right\}, \tag{38}$$

where $W(J, \bar{g})$ is the generating functional of connected Green functions. In (38), the DeWitt notations are

其中 $W(J, \bar{g})$ 是连通格林函数的生成泛函。(38) 中使用了德维特记号

$$J\phi = \int dx J_i(x) \phi^i(x), \quad \text{where} \quad J_i(x) = \left\{J^{\mu\nu}(x), J_\alpha^{(B)}(x), \bar{J}_\alpha(x), J_\alpha(x)\right\} \tag{39}$$

are external sources for the fields (21). The Grassmann parities and ghost numbers of these sources satisfy the relations

是场 (21) 的外源。这些源的格拉斯曼奇偶性和鬼数满足关系

$$\varepsilon\left(J_i\right)=\varepsilon\left(\phi^i\right),\ \operatorname{gh}\left(J_i\right)=\operatorname{gh}\left(\phi^i\right). \tag{40}$$

Let us start a detailed analysis of the generating functionals and their gauge dependencies from the generating functional (38). Later on, to complete the program to prove gauge invariance of renormalization in QG, we shall introduce a more general object $Z\left(J,\phi^*,\bar{g}\right)$, additionally depending on the antifields ϕ^* . This extended definition will be given below, and now we work out a more simple case, just to introduce the necessary notions in a more transparent way.

我们从生成泛函 (38) 开始，详细分析生成泛函及其规范依赖性。后续，为完成证明量子引力中重整化规范不变性的纲领，我们将引入更一般的对象 $Z\left(J,\phi^*,\bar{g}\right)$ ，它额外依赖于反场 ϕ^* 。我们会在下文给出这个推广定义，现在我们先讨论更简单的情况，以更清晰地引入必要的概念。

As a first step, consider the vacuum functional $Z_\Psi\left(\bar{g}\right)$, which corresponds to the choice of the gauge-fixing functional (29) in the presence of external metric $\bar{g}$,

第一步，考虑真空泛函 $Z_\Psi\left(\bar{g}\right)$ ，它对应存在外度规 $\bar{g}$ 时规范固定泛函 (29) 的选取，

$$Z_\Psi\left(\bar{g}\right)=\int d\phi\exp\left\{\frac{i}{\hbar}\left[S_0\left(\bar{g}+h\right)+\Psi\left(\phi,\bar{g}\right)\widehat{R}\left(\phi,\bar{g}\right)\right]\right\}=\exp\left\{\frac{i}{\hbar}W_\Psi\left(\bar{g}\right)\right\}, \tag{41}$$

where we introduced the operator

其中我们引入了算符

$$\widehat{R}\left(\phi,\bar{g}\right)=\int dx\frac{\delta_r}{\delta\phi^i\left(x\right)}R^i\left(x;\phi,\bar{g}\right) \tag{42}$$

and the fermionic gauge-fixing functional $\Psi\left(\phi,\bar{g}\right)$,

以及费米型规范固定泛函 $\Psi\left(\phi,\bar{g}\right)$,

$$\Psi\left(\phi,\bar{g}\right)=\int dx\sqrt{-\bar{g}\left(x\right)}\bar{C}^\alpha\chi_\alpha\left(x;\bar{g},h\right). \tag{43}$$

Taking into account (42) and (43), the expression in the exponential in the integrand of (41) is nothing but $\frac{i}{\hbar}S_{FP}\left(\phi,\bar{g}\right)$. Thus, the expression (41) becomes

考虑到 (42) 和 (43)，(41) 被积函数中指数上的表达式正是 $\frac{i}{\hbar}S_{FP}\left(\phi,\bar{g}\right)$ 。因此，(41) 可写为

$$Z_\Psi\left(\bar{g}\right)=\int d\phi\exp\left\{\frac{i}{\hbar}S_{FP}\left(\phi,\bar{g}\right)\right\}, \tag{44}$$

which is just (38) without the source term in the exponential.

这正是不含指数中源项的式 (38)。

The main advantage of using the fermionic gauge-fixing functional $\Psi(\phi,\bar{g})$ is that it is a scalar function that contains the information about the gauge-fixing function $\chi_\alpha(x;\bar{g},h)$. In particular, the possible change of the gauge fixing can be explored by evaluating $Z_{\Psi+\delta\Psi}$, which is the modified vacuum functional corresponding to $\Psi(\phi,\bar{g})+\delta\Psi(\phi,\bar{g})$. Here, $\delta\Psi(\phi,\bar{g})$ is an infinitesimal functional with odd Grassmann parity. Besides this requirement, $\delta\Psi(\phi,\bar{g})$ maybe an arbitrary variation corresponding to the change of $\chi_\alpha(x;\bar{g},h)$ in Eq. (43).

使用费米子规范固定泛函 $\Psi(\phi,\bar{g})$ 的主要优势在于，它是一个包含规范固定函数 $\chi_\alpha(x;\bar{g},h)$ 全部信息的标量函数。具体而言，我们可以通过计算对应于 $\Psi(\phi,\bar{g})+\delta\Psi(\phi,\bar{g})$ 的修正真空泛函 $Z_{\Psi+\delta\Psi}$，来研究规范固定可能发生的变化。此处 $\delta\Psi(\phi,\bar{g})$ 是具有奇数格拉斯曼奇偶性的无穷小泛函。除这一要求外，$\delta\Psi(\phi,\bar{g})$ 可以是对应式 (43) 中 $\chi_\alpha(x;\bar{g},h)$ 变化的任意变分。

Taking into account (44), with the new term, we get the vacuum functional with the variation of the gauge fixing:

考虑到式 (44)，加入新项后我们得到规范固定变分下的真空泛函:

$$Z_{\Psi+\delta\Psi}(\bar{g}) = \int d\phi \exp\left\{\frac{i}{\hbar}\left[S_{FP}(\phi,\bar{g}) + \delta\Psi(\phi,\bar{g})\,\widehat{R}(\phi,\bar{g})\right]\right\}. \tag{45}$$

The next step is to make the change of quantum variables ϕ^i in the form of BRST transformations (23), but with replacing the constant parameter μ by a functional $\mu=\mu(\phi,\bar{g})$,

下一步是对量子变量 ϕ^i 按 BRST 变换 (23) 的形式做变换,只需将常数参数 μ 替换为泛函 $\mu=\mu(\phi,\bar{g})$,

$$\phi^i(x) \to \phi'^i(x) = \phi^i(x) + R^i(x;\phi,\bar{g})\,\mu(\phi,\bar{g}) = \phi^i(x) + \Delta\phi^i(x). \tag{46}$$

In what follows, we shall use compact notations for the generators $R^i(x;\phi,\bar{g}) = R^i(x)$ and for the functional $\mu(\phi,\bar{g})=\mu$. Owing to the linearity of BRST transformations and the absence of derivatives acting on the parameter μ , the total action $S_{FP}(\phi,\bar{g})$ remains invariant under (46) even for the nonconstant μ . It is easy to check that the Jacobian of transformations (46) reads [51] (Let us note that the Jacobian of the transformations (46) can be evaluated exactly [52,53] but we will not reproduce this calculation here)

接下来，我们对生成元 $R^i(x;\phi,\bar{g}) = R^i(x)$ 和泛函 $\mu(\phi,\bar{g})=\mu$ 采用紧凑记号。由于 BRST 变换是线性的，且不存在作用在参数 μ 上的导数，因此即使 μ 不是常数，总作用量 $S_{FP}(\phi,\bar{g})$ 在 (46) 下仍然保持不变。不难验证，变换 (46) 的雅可比行列式为 [51](需要说明的是，变换 (46) 的雅可比行列式可以精确计算 [52,53]，本文不复现该计算过程)

$$J = J(\phi,\bar{g}) = \exp\left\{\int dx (-1)^{\varepsilon_i} M^i_i(x,x)\right\}, \tag{47}$$

where matrix $M^i{}_j(x,y)$ has the form

其中矩阵 $M^i{}_j(x,y)$ 形式为

$$M^i_j(x,y)=\frac{\delta_r\Delta\phi^i(x)}{\delta\phi^j(y)}=(-1)^{\varepsilon_j+1}\frac{\delta_r\mu}{\delta\phi^j(y)}R^i(x)-(-1)^{\varepsilon_j(\varepsilon_i+1)}\frac{\delta_l R^i(x)}{\delta\phi^j(y)}\mu.$$

(48)

In the theories of the Yang-Mills type because of the antisymmetry of the structure constants, there is the following relation:

在杨-米尔斯类型的理论中，由于结构常数具有反对称性，存在如下关系:

$$\int dx(-1)^{\varepsilon_i}\frac{\delta_l R^i(x)}{\delta\phi^i(x)}=0. \tag{49}$$

This relation can be verified using the definitions (25). Using (49), from (47) and (48), it follows that

该关系可以利用定义 (25) 验证。利用式 (49)，由式 (47) 和 (48) 可得

$$J=\exp\{-\mu(\phi,\bar{g})\widehat{R}(\phi,\bar{g})\}, \tag{50}$$

where we used the operator (42).

此处我们使用了式 (42) 中的算符。

Choosing the functional μ in the form

将泛函 μ 取为如下形式

$$\mu=\frac{i}{\hbar}\delta\Psi(\phi,\bar{g}), \tag{51}$$

one can observe that the described change of variables in the functional integral completely compensates the modification in the expression (45) compared to the initial formula (44). Thus, we arrive at the gauge independence of the vacuum functional

可以发现，泛函积分中上述变量变换完全抵消了式 (45) 相对初始公式 (44) 的修正。因此我们得到真空泛函的规范无关性

$$Z_\Psi(\bar{g})=Z_{\Psi+\delta\Psi}(\bar{g}). \tag{52}$$

The last identity can be written as the vanishing variation of the vacuum functionals Z or for W, as introduced in (41):

上一恒等式可以写为真空泛函 Z 的变分为零，或对应 (41) 中引入的 W 的变分为零:

$$\delta_\psi Z(\bar{g})=0\ \text{ or }\ \delta_\psi W(\bar{g})=0. \tag{53}$$

The invariance (52) shows that we can omit the label Ψ in the definition of the generating functionals (38).

不变性 (52) 表明，我们可以在生成泛函 (38) 的定义中省略标注 Ψ 。

The importance of the invariance (52) is related to the equivalence theorem by Kallosh and Tyutin [54]. This QFT theorem states that if the vacuum functional is invariant, the S -matrix does not depend on the gauge-fixing choice (see also [55, 56]). In the QG case, the existence of the S -matrix requires that the background metric $\bar{g}_{\mu\nu}$ admits asymptotic states. In particular, this requirement is satisfied if there is a flat Minkowski metric in some extremes, such that, in these regions, one can form asymptotic in and out states for the gravitons or, more generally, for the gravitational perturbations over the flat space-time. If this condition is satisfied, the invariance (52) implies that the S -matrix in the QG theory does not depend on the gauge fixing. One can say that if the theory and the physical situation to which this theory is applied admit the construction of the S -matrix, the last will be independent on the choice of the gauge-fixing conditions.

不变性 (52) 的重要性与 Kallosh 和 Tyutin 提出的等价定理 [54] 相关。该量子场论定理指出，若真空泛函具有不变性，则 S 矩阵不依赖于规范固定的选择 (另见 [55, 56])。在量子引力情形下，S 矩阵的存在要求背景度规 $\bar{g}_{\mu\nu}$ 允许渐近态存在。若空间在某些极限下为平直闵氏度规，该条件即可满足: 在这些区域中，我们可以形成引力子的渐进入态、渐进出态，更一般地说，可以形成平直时空上引力扰动的渐进态。若该条件满足，不变性 (52) 意味着量子引力理论中的 S 矩阵不依赖于规范固定。可以说，如果理论以及该理论所应用的物理场景允许构造 S 矩阵，那么该矩阵就不依赖于规范固定条件的选择。

It is remarkable that we can make such a strong statement for an arbitrary model of QG, even without requiring renormalizability of the theory or the locality of the classical action. Let us stress that the statement formulated above is valid only within the conventional perturbative approach to QFT or to the QG, as particular case. On the other hand, the situation may be completely different in other approaches. The discussion of these situations (including the asymptotic safety scenario in QG) and further references can be found in Ref. [38].

值得注意的是，我们能够对任意量子引力模型得出如此强有力的结论，甚至不需要该理论可重整化，也不需要经典作用量满足定域性。需要强调的是，上述结论仅在量子场论的传统微扰方法中成立——作为特例，对量子引力也同样如此。另一方面，在其他研究方法中情况可能完全不同。关于这些情况 (包括量子引力中的渐近安全情景) 的讨论和更多参考文献可以在文献 [38] 中找到。

On another hand, in QG gravity, the S -matrix can be hardly seen as the object of central interest. One of the reasons is that the gravitational fields in the asymptotic states should be free and this cannot be provided in the framework of general relativity. On the other hand, the main applications of QG are assumed in cosmology and black hole physics. And in both cases, there are no direct relations to the scattering problems, and the S -matrix does not look the most appropriate object to explore. Taking these points into account, it may be more advantageous to work with the effective action, taking control of its non-universality, i.e., the dependence on the gauge fixing and, more generally, on the parametrization of quantum fields. Let us now see how this can be done.

另一方面，在量子引力中，S 矩阵很难被当作核心研究对象。原因之一在于，渐近态中的引力场应当是自由的，而广义相对论的框架无法满足这一点。此外，量子引力的主要应用被认为是在宇宙学和黑洞物理中，而这两类场景都和散射问题没有直接关联，因此 S 矩阵看起来不是最合适的研究对象。考虑到这些因素，研究有效作用量可能更有优势，同时我们需要把控它的非普适性，也就是它对规范固定的依赖，更一般地说，是对量子场参数化的依赖。下面我们就来看这一工作如何开展。

The effective action $\Gamma(\Phi, \bar{g})$ is defined by means of Legendre transformation:

有效作用量 $\Gamma(\Phi, \bar{g})$ 通过勒让德变换定义如下:

$$\Gamma(\Phi, \bar{g}) = W(J, \bar{g}) - J_i \Phi^i, \tag{54}$$

where $\Phi = \left\{\Phi^i\right\}$ are mean fields and J_i are the solutions of the equations

其中 $\Phi = \left\{\Phi^i\right\}$ 是平均场，J_i 是下述方程的解

$$\frac{\delta W(J, \bar{g})}{\delta J_i} = \Phi^i \quad \text{and} \quad J_i \Phi^i = \int dx J_i(x) \Phi^i(x). \tag{55}$$

In terms of effective action, the property (53) means the on-shell independence on the gauge-fixing condition and can be cast in the form

用有效作用量表述的话，性质 (53) 意味着壳上规范固定条件无关性，可以写成如下形式

$$\delta_\Psi \Gamma(\Phi, \bar{g})\Big|_{\frac{\delta\Gamma(\Phi,\bar{g})}{\delta\Phi}=0} = 0, \tag{56}$$

i.e., the effective action evaluated on its extremal does not depend on gauge.

也就是说，在极值处计算得到的有效作用量不依赖于规范选择。

Until now, we did not assume that the background metric may transform under the general coordinate transformation. This was a necessary approach, as we mentioned after the definition of the splitting (13) of the metric into the background and quantum parts. However, when the effective action is defined, we can perform the coordinate transformation of the background metric $\bar{g}_{\mu\nu}$, together with the corresponding transformation for the quantum metric. Taking a variation of the background metric $\bar{g}_{\mu\nu}$ under general coordinate transformations, we get

到目前为止，我们还没有假设背景度规可以在广义坐标变换下变换。正如我们在定义 (13) 将度规拆分为背景部分和量子部分之后提到的，这是必要的处理方式。但在有效作用量定义完成后，我们可以对背景度规 $\bar{g}_{\mu\nu}$ 执行坐标变换，同时对量子度规做相应的变换。对背景度规 $\bar{g}_{\mu\nu}$ 在广义坐标变换下取变分，我们得到

$$\delta^{(c)}_\omega \bar{g}_{\mu\nu} = R_{\mu\nu\sigma}(\bar{g})\, \omega^\sigma. \tag{57}$$

Here, the symbol (c) indicates that the transformation concerns the background metric, i.e., it is performed in the sector of classical fields. It is important that this transformation does not lead neither to the change of the form of the Faddeev-Popov action (16) nor to the change of the transformation rules for the auxiliary and ghost fields.

此处符号 (c) 表明该变换针对背景度规，即它是在经典场扇区中执行的。重要的是，这个变换既不会改变法捷耶夫-波波夫作用量 (16) 的形式，也不会改变辅助场和鬼场的变换规则。

In the quantum fields sector $h_{\mu\upsilon}$, the form of the transformations is fixed by requiring the invariance of the action:

在量子场扇区 $h_{\mu\upsilon}$ 中，变换的形式由作用量的不变性要求确定:

$$\delta_{\omega}^{(q)} h_{\mu\nu} = R_{\mu\nu\sigma}(h)\,\omega^{\sigma} = -\omega^{\sigma}\partial_{\sigma}h_{\mu\nu} - h_{\mu\sigma}\partial_{\nu}\omega^{\sigma} - h_{\sigma\nu}\partial_{\mu}\omega^{\sigma}, \tag{58}$$

where the symbol (q) indicates the gauge transformations of quantum fields. For the action, we have

其中符号 (q) 表示量子场的规范变换。对于作用量，我们有

$$\delta_{\omega} S_0(\bar{g}+h) = 0, \ \delta_{\omega} = \delta_{\omega}^{(c)} + \delta_{\omega}^{(q)}. \tag{59}$$

With these definitions, for the variation of $Z(\bar{g})$, we have

根据这些定义，对 $Z(\bar{g})$ 做变分可得

$$\delta_{\omega}^{(c)} Z(\bar{g}) = \frac{i}{\hbar}\int d\phi \left[\delta_{\omega}^{(c)} S_0(\bar{g}+h) + \delta_{\omega}^{(c)} S_{gh}(\phi,\bar{g}) + \delta_{\omega}^{(c)} S_{gf}(\phi,\bar{g})\right]$$

$$\times \exp\left\{\frac{i}{\hbar} S_{FP}(\phi,\bar{g})\right\}. \tag{60}$$

Let us stress that, for a while, we consider only the transformations of $\bar{g}$, such that $\delta^{(q)}$ does not enter the last expression. At the next stage, using a change of variables in the functional integral (60), we have to arrive at the relation $\delta_{\omega}^{(c)} Z(\bar{g}) = 0$, that is, to prove invariance of $Z(\bar{g})$ under the transformations (57).

需要强调的是，目前我们仅考虑 $\bar{g}$ 的变换，因此 $\delta^{(q)}$ 不会出现在最后这个表达式中。在下一阶段，我们利用泛函积分 (60) 中的变量替换，最终会得到关系 $\delta_{\omega}^{(c)} Z(\bar{g}) = 0$ ，也就证明了 $Z(\bar{g})$ 在变换 (57) 下的不变性。

The gauge-fixing action $S_{gf}(\phi,\bar{g})$ depends only on the three field variables $h_{\mu\upsilon}$, B^{α} , and $\bar{g}_{\mu\nu}$. For $h_{\mu\nu}$ and $\bar{g}_{\mu\nu}$, the transformation law has been already defined in (57) and (58). Thus, we need to define the transformation for the remaining field B^{α} . The new rule $\delta_{\omega}^{(q)} B^{\alpha}$ should compensate the variation of $S_{gf}(\phi,\bar{g})$ caused by the transformations of $\bar{g}_{\mu\nu}$ and $h_{\mu\nu}$. The corresponding condition has the form

规范固定作用量 $S_{gf}(\phi,\bar{g})$ 仅依赖三个场变量 $h_{\mu\upsilon}$、B^{α} 和 $\bar{g}_{\mu\nu}$。对于 $h_{\mu\nu}$ 和 $\bar{g}_{\mu\nu}$，变换规则已在 (57) 和 (58) 中定义，因此我们只需定义剩余场 B^{α} 的变换。新规则 $\delta_{\omega}^{(q)}B^{\alpha}$ 应当抵消 $\bar{g}_{\mu\nu}$ 和 $h_{\mu\nu}$ 的变换引起的 $S_{gf}(\phi,\bar{g})$ 变分，对应的条件形式为

$$\delta_{\omega}S_{gf} = \int dx\sqrt{-\bar{g}}\left[\left(\delta_{\omega}^{(q)}B^{\alpha} + \omega^{\sigma}\partial_{\sigma}B^{\alpha}\right)\chi_{\alpha}(\bar{g},h)\right.$$

$$\left. + B^{\alpha}\omega^{\sigma}\partial_{\sigma}\chi_{\alpha}(\bar{g},h) + B^{\alpha}\delta_{\omega}\chi_{\alpha}(\bar{g},h)\right]. \tag{61}$$

The transformation of the gauge-fixing functions χ_{α} cannot be defined independently since they are constructed from the metric and the last transforms according to Eq. (57). Thus, the variation of the gauge-fixing functions χ_{α} has the form of the vector field transformation (6):

规范固定函数 χ_{α} 的变换无法独立定义，因为它们由度规构造，而度规的变换已由式 (57) 给出。因此，规范固定函数 χ_{α} 的变分具有向量场变换 (6) 的形式:

$$\delta_{\omega}\chi_{\alpha} = -\omega^{\sigma}\partial_{\sigma}\chi_{\alpha} - \chi_{\sigma}\partial_{\alpha}\omega^{\sigma}. \tag{62}$$

It its turn, the transformation of the auxiliary field B can be chosen according to the same vector rule (7). This gives

相应地，辅助场 B 的变换可以按相同的向量规则 (7) 选取，由此得到

$$\delta_{\omega}^{(q)}B^{\alpha} = -\omega^{\sigma}\partial_{\sigma}B^{\alpha} + B^{\sigma}\partial_{\sigma}\omega^{\alpha}. \tag{63}$$

It is easy to check that (62) and (63) provide the desired invariance in (61),

不难验证，(62) 和 (63) 给出了 (61) 中要求的不变性,

$$\delta_{\omega}S_{gf} = 0. \tag{64}$$

In the same way, one can confirm the invariance of the ghost action:

同理可以证明显鬼作用量的不变性:

$$\delta_{\omega}S_{gh} = 0 \tag{65}$$

for the vector transformation laws for the ghost fields $\bar{C}^{\alpha}$ and C^{α} :

对于鬼场 $\bar{C}^{\alpha}$ 和 C^{α} 的向量变换规则:

$$\delta_{\omega}^{(q)}\bar{C}^{\alpha}(x) = -\omega^{\sigma}(x)\partial_{\sigma}\bar{C}^{\alpha}(x) + \bar{C}^{\rho}\partial_{\rho}\omega^{\alpha}(x),$$

$$\delta_{\omega}^{(q)}C^{\alpha}(x) = -\omega^{\sigma}(x)\partial_{\sigma}C^{\alpha}(x) + C^{\rho}\partial_{\rho}\omega^{\alpha}(x). \tag{66}$$

All in all, we can conclude that the total Faddeev-Popov action S_{FP} is invariant

综上，我们可以得出结论: 总法捷耶夫-波波夫作用量 S_{FP} 不变

$$\delta_\omega S_{FP} = 0 \tag{67}$$

under the new version of gauge transformations, which is based on the background transformations of quantum fields ϕ and of $\bar{g}$, i.e.,(57),(58),(63), and (66).

在新版规范变换下成立，该变换基于量子场 ϕ 和 $\bar{g}$ 的背景变换，即 (57)、(58)、(63) 和 (66)。

As a consequence of (67), important property (44), and the invariance of the integration measure, the vacuum functional possesses gauge invariance:

由 (67)、性质 (44) 以及积分测度的不变性可得，真空泛函具有规范不变性:

$$\delta_\omega Z(\bar{g}) = \delta_\omega^{(c)} Z(\bar{g}) = 0. \tag{68}$$

As we shall see in what follows, one can use Eq. (68) to prove the gauge invariance of an important object called the background effective action, i.e., the effective action with the switched off mean fields, $\Gamma(\bar{g}) = \Gamma(\Phi = 0, \bar{g})$. The required feature can be formulated as

我们接下来会看到，利用式 (68) 可以证明一个重要对象——背景有效作用量，即平均场 $\Gamma(\bar{g}) = \Gamma(\Phi = 0, \bar{g})$ 取零后的有效作用量——的规范不变性，所需性质可以表述为

$$\delta_\omega^{(c)} \Gamma(\bar{g}) = 0. \tag{69}$$

Let us note that switching off the mean quantum fields $\Phi = \{h, C, \bar{C}, B\}$ requires special care, and we shall see how this should be done. Then, the resulting relation (69) is one of the main targets of our consideration.

请注意，关闭平均量子场 $\Phi = \{h, C, \bar{C}, B\}$ 需要特别处理，我们会说明正确的处理方法。由此得到的关系式 (69) 是我们研究的核心目标之一。

It is useful to start by exploring the off-shell gauge invariance of the generating functionals of our interest. To this end, it is useful to present the background transformations (57), (58), (63), and (66) in the form

我们不妨从探究我们所关注的生成泛函的脱壳规范不变性入手。为此，我们可以将背景变换 (57)、(58)、(63) 和 (66) 写为如下形式

$$\delta_\omega^{(c)} \bar{g}_{\mu\nu} = R_{\mu\nu\sigma}(\bar{g})\,\omega^\sigma, \;\; \delta_\omega^{(q)} \phi^i = \mathcal{R}^i_\sigma(\phi)\,\omega^\sigma, \tag{70}$$

where the generators $\mathcal{R}^i_\sigma(\phi)$ are linear in the quantum fields ϕ and do not depend on the background metric $\bar{g}$. Utilizing these notations, the general form of the transformation of an arbitrary functional $\Gamma = \Gamma(\phi, \bar{g})$ can be written in the form

其中生成元 $\mathcal{R}^i_\sigma(\phi)$ 对量子场 ϕ 是线性的，且不依赖于背景度规 $\bar{g}$ 。利用这些记号，任意泛函 $\Gamma = \Gamma(\phi, \bar{g})$ 变换的一般形式可以写为

$$\delta_\omega \Gamma = \delta^{(c)}_\omega \Gamma + \frac{\delta_r \Gamma}{\delta \phi^i} \mathcal{R}^i_\sigma(\phi)\, \omega^\sigma. \tag{71}$$

Consider the variation of the generating functional $Z(J, \bar{g})$ (38) , under the gauge transformations of the background metric

我们来考察背景度规规范变换下生成泛函 $Z(J, \bar{g})$ (38) 的变分

$$\delta^{(c)}_\omega Z(J, \bar{g}) = \frac{i}{\hbar} \int d\phi \delta^{(c)}_\omega S_{FP}(\phi, \bar{g}) \exp\left\{\frac{i}{\hbar}\left[S_{FP}(\phi, \bar{g}) + J\phi\right]\right\}. \tag{72}$$

Using the background transformations in the sector of quantum fields ϕ and taking into account that for the linear change of variables the Jacobian of this transformation is independent on the fields, we arrive at the relation

利用量子场 ϕ 部分的背景变换，同时考虑到对于线性变量变换，变换的雅可比行列式与场无关，我们可以得到如下关系式

$$\frac{i}{\hbar} \int d\phi \left\{\delta^{(q)}_\omega S_{FP}(\phi, \bar{g}) + J\delta^{(q)}_\omega \phi\right\} \exp\left\{\frac{i}{\hbar}\left[S_{FP}(\phi, \bar{g}) + J\phi\right]\right\} = 0. \tag{73}$$

On the other hand, from (67) and (73), it follows that

另一方面，由 (67) 和 (73) 可以推出

$$\begin{aligned}\delta^{(c)}_\omega Z(J, \bar{g}) &= \frac{i}{\hbar} \int d\phi J_j \mathcal{R}^j_\sigma(\phi)\, \omega^\sigma \exp\left\{\frac{i}{\hbar}\left[S_{FP}(\phi, \bar{g}) + J\phi\right]\right\},\\ &= \frac{i}{\hbar} J_j \mathcal{R}^j_\sigma\left(\frac{\hbar}{i}\frac{\delta}{\delta J}\right) Z(J, \bar{g})\, \omega^\sigma.\end{aligned} \tag{74}$$

In terms of the generating functional $W = W(J, \bar{g}) = -i\hbar \ln Z(J, \bar{g})$ of connected Green functions, the relation (74) reads

对连通格林函数的生成泛函 $W = W(J, \bar{g}) = -i\hbar \ln Z(J, \bar{g})$ ，关系式 (74) 可以写为

$$\delta^{(c)}_\omega W(J, \bar{g}) = J_j \mathcal{R}^j_\sigma\left(\frac{\delta W}{\delta J}\right) \omega^\sigma, \tag{75}$$

where we used linearity of generators $\mathcal{R}^i_\sigma(\phi)$ with respect to ϕ . Now, we can consider the generating functional of vertex functions (the effective action):

其中我们用到了生成元 $\mathcal{R}^i_\sigma(\phi)$ 关于 ϕ 的线性性质。现在，我们可以考察顶角函数的生成泛函，即有效作用量:

$$\Gamma = \Gamma(\Phi, \bar{g}) = W(J, \bar{g}) - J\Phi, \tag{76}$$

$$\text{where } \Phi^j = \frac{\delta_l W}{\delta J_j}, \ \frac{\delta_r \Gamma}{\delta \Phi^j} = -J_j \ \text{ and } \ \delta W = \delta \Gamma\,. \tag{77}$$

Taking the variation of external metric and the mean fields (70), in terms of Γ , the relation (75) becomes

对外部度规和平均场 (70) 取变分，用 Γ 表示后，关系式 (75) 变为

$$\delta^{(c)}_{\omega}\Gamma(\Phi, \bar{g}) = -\frac{\delta_r \Gamma}{\delta \Phi^j}\mathcal{R}^j_{\sigma}(\Phi)\,\omega^{\sigma}. \tag{78}$$

Finally, taking into account the variations of all fields (70) and using the identity (71), we arrive at

最后，考虑所有场 (70) 的变分并利用恒等式 (71)，我们得到

$$\delta_{\omega}\Gamma(\Phi, \bar{g}) = 0. \tag{79}$$

It turns out that the relations (78) and (79) prove the fundamental property (69). In order to see this, one has to note that the generators of quantum fields (58), (66), and (63) have linear dependence of these fields. As a result, one meets the following limit for the generators $\mathcal{R}^i_{\sigma}(\Phi)$ when the mean fields are switched off:

可以证明，关系式 (78) 和 (79) 证明了基本性质 (69)。需要注意的是，量子场的生成元 (58)、(66) 和 (63) 对这些场是线性依赖的。因此，当平均场关闭时，生成元 $\mathcal{R}^i_{\sigma}(\Phi)$ 满足如下极限:

$$\lim_{\Phi\to 0}\mathcal{R}^j_{\sigma}(\Phi) = 0. \tag{80}$$

This relation shows that Γ is invariant under nondeformed background transformations, i.e., possesses the same invariance as the classical Faddeev-Popov action.

该关系式表明 Γ 在非形变背景变换下不变，即拥有与经典法捷耶夫-波波夫作用量相同的不变性。

In the renormalization program based on Batalin-Vilkovisky formalism, the extended action is $S = S(\phi, \phi^*, \bar{g})$ defined in (30). The precursors for the full effective action are the extended generating functional of Green functions $Z = Z(J, \phi^*, \bar{g})$ and of the connected Green functions $W = W(J, \phi^*, \bar{g})$:

在基于巴达林-维尔科夫斯基形式的重整化方案中，推广作用量是由 (30) 定义的 $S = S(\phi, \phi^*, \bar{g})$ 。完全有效作用量的前驱是格林函数的推广生成泛函 $Z = Z(J, \phi^*, \bar{g})$ 和连通格林函数的推广生成泛函 $W = W(J, \phi^*, \bar{g})$:

$$Z(J, \phi^*, \bar{g}) = \int d\phi \exp\Big\{\frac{i}{\hbar}[S(\phi, \phi^*, \bar{g}) + J\phi]\Big\} = \exp\Big\{\frac{i}{\hbar}W(J, \phi^*, \bar{g})\Big\}. \tag{81}$$

Our first purpose is to prove that the effective action satisfies the quantum version of Eq. (37). Due to the invariance of S_{FP} under background field transformations, the variation of S takes the form

我们的首要目标是证明有效作用量满足式 (37) 的量子形式。由于 S_{FP} 在背景场变换下不变，S 的变分可以写为

$$\delta_\omega S(\phi, \phi^*, \bar{g}) = \phi_i^* \delta_\omega R^i(\phi, \bar{g}), \tag{82}$$

that shows that the action is gauge invariant on the hypersurface $\phi_i^* = 0$.

这表明作用量在超曲面 $\phi_i^* = 0$ 上是规范不变的。

Using the condensed DeWitt's notation, one can write the variations of the generators $\delta_\omega R^i(\phi, \bar{g})$ in the following compact form:

使用德维特的缩记法，我们可以将生成元 $\delta_\omega R^i(\phi, \bar{g})$ 的变分写为如下紧凑形式:

$$\delta_\omega R^{(h)}_{\mu\nu}(\phi, \bar{g}) = -\omega^\sigma \partial_\sigma R_{\mu\nu\lambda}(\bar{g} + h) C^\lambda - \partial_\mu \omega^\sigma R_{\sigma\nu\lambda}(\bar{g} + h) C^\lambda$$

$$-\partial_\nu \omega^\sigma R_{\mu\sigma\lambda}(\bar{g} + h) C^\lambda$$

$$\delta_\omega R^\alpha_{(B)}(\phi, \bar{g}) = 0$$

$$\delta_\omega R^\alpha_{(C)}(\phi, \bar{g}) = \omega^\sigma \partial_\sigma \left(C^\lambda \partial_\lambda C^\alpha\right) - C^\lambda \partial_\lambda C^\sigma \partial_\sigma \omega^\alpha,$$

$$\delta_\omega R^\alpha_{(\bar{C})}(\phi, \bar{g}) = -\omega^\sigma \partial_\sigma B^\alpha + B^\sigma \partial_\sigma \omega^\alpha. \tag{83}$$

The variations $\delta_\omega R^i(\phi, \bar{g})$ are at most quadratic in the sector of fields $h_{\mu\nu}$ and C^α and linear in the field $\bar{C}^\alpha$.

变分 $\delta_\omega R^i(\phi, \bar{g})$ 在场 $h_{\mu\nu}$ 和 C^α 的区域中至多为二阶，在场 $\bar{C}^\alpha$ 中为一阶。

Let us now consider the variation of the extended generating functional $Z(J, \phi^*, \bar{g})$ (81) under the gauge transformations of external metric $\bar{g}$:

现在我们考虑扩展生成泛函 $Z(J, \phi^*, \bar{g})$ (81) 在外度规 $\bar{g}$ 的规范变换下的变分:

$$\delta^{(c)}_\omega Z(J, \phi^*, \bar{g}) = \frac{i}{\hbar} \int d\phi \left\{\delta^{(c)}_\omega S_{FP}(\phi, \bar{g}) + \phi_i^* \delta^{(c)}_\omega R^i(\phi, \bar{g})\right\}$$

$$\times \exp\left\{\frac{i}{\hbar}\left[S(\phi, \phi^* \bar{g}) + J\phi\right]\right\}. \tag{84}$$

Making the change of variables ϕ^i according to (58),(63), and (66) in the functional integral and taking into account the triviality of the corresponding Jacobian, we arrive at the relation

根据式 (58)、(63) 和 (66) 对泛函积分中的变量 ϕ^i 做换元，并考虑到相应雅可比行列式是平凡的，我们得到关系

$$\frac{i}{\hbar} \int d\phi \left\{\delta^{(q)}_\omega S_{FP}(\phi, \bar{g}) + \phi_i^* \delta^{(q)}_\omega R^i(\phi, \bar{g}) + J_i \delta^{(q)}_\omega \phi^i\right\}$$

$$\times \exp\left\{\frac{i}{\hbar}\left[S\left(\phi,\phi^{*}\bar{g}\right)+J\phi\right]\right\}=0. \tag{85}$$

Combining Eqs. (84) and (85) and using the invariance of S_{FP} (67), we obtain

合并式 (84) 和 (85)，并利用 S_{FP} (67) 的不变性，我们得到

$$\delta_{\omega}^{(c)}Z\left(J,\phi^{*},\bar{g}\right)=\frac{i}{\hbar}\int d\phi\left\{\phi_{i}^{*}\delta_{\omega}R^{i}\left(\phi,\bar{g}\right)+J_{i}\mathcal{R}_{\sigma}^{i}\left(\phi\right)\omega^{\sigma}\right\}$$

$$\times \exp\left\{\frac{i}{\hbar}\left[S\left(\phi,\phi^{*}\bar{g}\right)+J\phi\right]\right\}, \tag{86}$$

or, equivalently,

或者等价地,

$$\delta_{\omega}^{(c)}Z\left(J,\phi^{*},\bar{g}\right)=\frac{i}{\hbar}\phi_{i}^{*}\delta_{\omega}R^{i}\left(\frac{\hbar}{i}\frac{\delta}{\delta J},\bar{g}\right)Z\left(J,\phi^{*},\bar{g}\right)$$

$$+\frac{i}{\hbar}J_{i}\mathcal{R}_{\sigma}^{i}\left(\frac{\hbar}{i}\frac{\delta}{\delta J}\right)Z\left(J,\phi^{*},\bar{g}\right)\omega^{\sigma}. \tag{87}$$

In terms of the generating functional of connected Green functions, (87) becomes

用连通格林函数的生成泛函表示，(87) 式变为

$$\delta_{\omega}^{(c)}W\left(J,\phi^{*},\bar{g}\right)=\phi_{i}^{*}\delta_{\omega}R^{i}\left(\frac{\delta W}{\delta J}+\frac{\hbar}{i}\frac{\delta}{\delta J},\bar{g}\right)\mathbf{1}+J_{i}\mathcal{R}_{\sigma}^{i}\left(\frac{\delta W}{\delta J}\right)\omega^{\sigma}, \tag{88}$$

where the symbol1means that the operator acts on the unit, $\mathbf{1}=1$. In the case of functional derivative, one has $\frac{\delta}{\delta\phi}\mathbf{1}=0$, but since in many cases the expressions are nonlinear, this is a useful notation.

其中符号 1 表示算子作用在单位元 $\mathbf{1}=1$ 上。对于泛函导数，有 $\frac{\delta}{\delta\phi}\mathbf{1}=0$ ，但由于在很多情况下表达式是非线性的，这是一个有用的记号。

The extended generating functional of vertex functions (extended effective action) is defined in a standard way. Starting from $W=W\left(J,\phi^{*},\bar{g}\right)$, introduced in Eq. (81), we perform Legendre transformation:

顶角函数的扩展生成泛函 (扩展有效作用量) 按标准方式定义。从式 (81) 引入的 $W=W\left(J,\phi^{*},\bar{g}\right)$ 出发，我们做勒让德变换:

$$\Gamma\left(\Phi,\phi^{*},\bar{g}\right)=W\left(J,\phi^{*},\bar{g}\right)-J\Phi,\ \Phi^{j}=\frac{\delta_{l}W}{\delta J_{j}},\ \frac{\delta_{r}\Gamma}{\delta\Phi^{j}}=-J_{j}. \tag{89}$$

One can easily see that this definition is a generalization of Eq. (54).

不难看出该定义是式 (54) 的推广。

As usual for the Legendre transformation, from (89), it follows that

对于勒让德变换，由 (89) 可得

$$\frac{\delta_r}{\delta J_k}\left(\frac{\delta_l W}{\delta J_i}\right)\times\frac{\delta_l}{\delta\Phi^i}\left(\frac{\delta_r\Gamma}{\delta\Phi^j}\right)=-\delta_j^k. \tag{90}$$

It proves useful to introduce the following notations:

引入以下记号是很有用的:

$$\delta_\omega\bar{R}^i\left(\Phi,\phi^*,\bar{g}\right)=\delta_\omega R^i\left(\widehat{\Phi},\bar{g}\right)\mathbf{1},\ \widehat{\Phi}^j=\Phi^j+i\hbar\left(\Gamma''^{-1}\right)^{jk}\frac{\delta_l}{\delta\Phi^k}, \tag{91}$$

where the symbol $(\Gamma''-1)^{jk}$ denotes the matrix inverse to

其中符号 $(\Gamma''-1)^{jk}$ 表示逆矩阵

$$(\Gamma'')_{ij}=\frac{\delta_l}{\delta\Phi^i}\left(\frac{\delta_r\Gamma}{\delta\Phi^j}\right)\ \text{ i.e., }\ \left(\Gamma''^{-1}\right)^{ik}\left(\Gamma''\right)_{kj}=\delta_j^i. \tag{92}$$

In terms of extended effective action (89), Eq. (88) rewrites as

用扩展有效作用量 (89) 表示，式 (88) 可改写为

$$\delta_\omega^{(c)}\Gamma\left(\Phi,\phi^*,\bar{g}\right)=-\frac{\delta_r\Gamma}{\delta\Phi^i}\mathcal{R}_\sigma^i\left(\Phi\right)\omega^\sigma+\phi_i^*\delta_\omega\bar{R}^i\left(\Phi,\phi^*,\bar{g}\right) \tag{93}$$

or, using the relation (71), in the form

或者利用关系 (71)，写成

$$\delta_\omega\Gamma\left(\Phi,\phi^*,\bar{g}\right)=\phi_i^*\delta_\omega\bar{R}^i\left(\Phi,\phi^*,\bar{g}\right). \tag{94}$$

At this point, we can draw a general conclusion concerning QG theories in the background field formalism. At the non-renormalized level, any covariant QG theory has the following general property: the extended quantum action $S=S\left(\phi,\phi^*,\bar{g}\right)$ satisfies the classical master (Zinn-Justin) equation of the Batalin-Vilkovisky formalism [49,50], as we already anticipated in Eq. (37). Furthermore, the extended effective action $\Gamma=\Gamma\left(\Phi,\phi^*,\bar{g}\right)$ has the same symmetries and therefore satisfies the same master equation:

至此，我们可以得到背景场形式下量子引力理论的一般性结论。在未重整化的层面上，任何协变量子引力理论都具备如下普遍性质: 扩展量子作用量 $S=S\left(\phi,\phi^*,\bar{g}\right)$ 满足巴塔林-维尔可夫斯基形式的经典主方程 (津恩-贾斯汀方程)[49,50]，正如我们在式 (37) 中已经预告的。此外，扩展有效作用量 $\Gamma=\Gamma\left(\Phi,\phi^*,\bar{g}\right)$ 具有相同的对称性，因此满足同一个主方程:

$$(\Gamma,\Gamma)=0. \tag{95}$$

According to Eq. (82), the functional $S=S\left(\phi,\phi^*,\bar{g}\right)$ and, according to (94), also $\Gamma=\Gamma\left(\Phi,\phi^*,\bar{g}\right)$ are invariant under the background gauge transformations on the hypersurface $\phi^*=0$:

根据式 (82)，泛函 $S = S(\phi, \phi^*, \bar{g})$ ，并根据式 (94)，还有 $\Gamma = \Gamma(\Phi, \phi^*, \bar{g})$ ，在超曲面 $\phi^* = 0$ 上的背景规范变换下都是不变的:

$$\delta_\omega S|_{\phi^*=0} = 0,\ \ \delta_\omega \Gamma|_{\phi^*=0} = 0 \tag{96}$$

and, more general, satisfy the relations (82) and (94).

并且，更一般地说，满足关系式 (82) 和 (94)。

Gauge Invariant Renormalizability

规范不变可重整性

Up to now, we were considering the non-renormalized generating functionals of Green functions. The next step is to prove the gauge invariant renormalizability, which is the property of renormalized generating functionals. In the framework of Batalin-Vilkovisky formalism, one can prove the BRST invariant renormalizability. The last means the preservation of basic equations (37) for the extended action $S(\phi, \phi^*, \bar{g})$ and an identical equation (95) for the extended effective action $\Gamma(\Phi, \Phi^*, \bar{g})$ after renormalization. The required identities for the classical and effective renormalized actions have the form

截至目前，我们讨论的都是未重整化的格林函数生成泛函。下一步是证明规范不变可重整性，这是重整化生成泛函的性质。在巴塔林-维尔可维斯基形式体系框架下，可以证明 BRST 不变可重整性。这意味着重整化后，推广作用量 $S(\phi, \phi^*, \bar{g})$ 的基本方程 (37) 和推广有效作用量 $\Gamma(\Phi, \Phi^*, \bar{g})$ 的恒等式 (95) 仍然成立。经典重整化作用量和有效重整化作用量所需满足的恒等式形式为

$$(S_R, S_R) = 0 \ \text{ and } \ (\Gamma_R, \Gamma_R) = 0. \tag{97}$$

Let us remember that the "classical" actions S and S_R are zero-order approximations of the loop expansions in the parameter $\hbar$ of the bare and renormalized effective actions Γ and Γ_R . In this sense, Eq. (37) is the zero-order approximation of Eq. (95) and what we have to do now is to extend these two equations to the renormalized quantities S_R and Γ_R . Our strategy will be to explore this extension order by order in the loop expansion parameter $\hbar$. In this way, we can prove that the renormalized actions S_R and Γ_R obey the gauge invariance property perturbatively.

我们回顾一下，" 经典" 作用量 S 和 S_R 分别是裸有效作用量 Γ 和重整化有效作用量 Γ_R 按圈展开参数 $\hbar$ 展开的零阶近似。在此意义上，式 (37) 是式 (95) 的零阶近似，我们现在需要做的是将这两个方程推广到重整化量 S_R 和 Γ_R 。我们的策略是按圈展开参数 $\hbar$ 逐阶推广这一关系。通过这种方法，我们可以证明重整化作用量 S_R 和 Γ_R 微扰地满足规范不变性性质。

BRST Invariant Renormalization

BRST 不变重整化

As it was explained above, our next purpose is to consider the loop expansion in the master equation (95) and arrive at the conclusion about the coordinate invariance of the n -loop approximation with switched-off sources. As a first step, consider the one-loop approximation for $\Gamma = \Gamma(\Phi, \Phi^*, \bar{g})$. To provide the uniformity of notations, we shall use $\Phi^* = \phi^*$ for the antifields.

正如上文所述，我们接下来的目的是研究主方程 (95) 中的圈展开，并得出关源后 n 圈近似坐标不变性的结论。第一步，我们考虑 $\Gamma = \Gamma(\Phi, \Phi^*, \bar{g})$ 的单圈近似。为统一符号，我们将反场记为 $\Phi^* = \phi^*$ 。

The effective action can be presented in the form

有效作用量可以写成如下形式

$$\Gamma = \Gamma^{(1)} + \mathcal{O}\left(\hbar^2\right) = S + \hbar\left[\Gamma^{(1)}_{\text{div}} + \Gamma^{(1)}_{\text{fin}}\right] + \mathcal{O}\left(\hbar^2\right), \tag{98}$$

where $S = S(\Phi, \Phi^*, \bar{g})$ is the tree-level action and $\Gamma^{(1)}_{\text{div}}$ and $\Gamma^{(1)}_{\text{fin}}$ stand for the divergent and finite parts of the one-loop approximation for Γ .

其中 $S = S(\Phi, \Phi^*, \bar{g})$ 是树图阶作用量，$\Gamma^{(1)}_{\text{div}}$ 和 $\Gamma^{(1)}_{\text{fin}}$ 分别对应 Γ 单圈近似的发散部分和有限部分。

In the local models of QG, the locality of the divergent part of effective action is guaranteed by Weinberg's theorem [57] (see also [58] for an alternative proof and further references). Let us note, by passing, that there is an exception from this general rule, corresponding to the theories with scalar field(s) and with a spontaneous symmetry breaking, in the presence of the background gravitational field. In this situation, there may be nonlocal divergences [59]; however, this can be seen as an exception that confirms the general rule instead of contradicting it.

在量子引力的定域模型中，有效作用量发散部分的定域性由温伯格定理保证 [57](替代证明和更多参考文献参见 [58])。顺便指出，该一般规则存在一个例外: 背景引力场存在时，带自发对称性破缺的标量场理论中可能出现非定域发散 [59]；但这一例外反而印证了一般规则，并不与之矛盾。

Furthermore, even if the starting action is nonlocal, the UV divergences are expected to be described by local functionals. The physical reason for this is that the high-energy domain always corresponds to the short-distance limit. And in the case of UV divergences, the energies are infinitely high; hence, the distances should be infinitely short, which does not leave space for the nonlocal divergences.

此外，即使初始作用量是非定域的，紫外发散也应当由定域泛函描述。其物理原因是: 高能区总是对应短距离极限；对于紫外发散，能量无穷大，因此距离必然无穷小，非定域发散没有存在的空间。

Since there is a special section of this handbook devoted to the nonlocal QG models, we will not extend this discussion and just mention references [21,22,60,61]. It was argued in these works that the UV divergent part of effective action, for a wide class of nonlocal models of QG, is local, including the ones with a nonlocal classical action. Taking this into account, we can assume that $\Gamma^{(1)}_{\text{div}}$ is a local functional. The divergence determines the form of the counterterms of the one-loop renormalized action, $\Delta S = -\hbar\Gamma^{(1)}_{\text{div}}$, such that the renormalized action is

由于本手册有专门章节介绍非定域量子引力模型，我们在此不展开讨论，仅列出参考文献[21,22,60,61]。这些工作指出，对于一大类非定域量子引力模型 (包括经典作用量本身非定域的情况)，有效作用量的紫外发散部分仍是定域的。据此我们可以假设 $\Gamma^{(1)}_{\text{div}}$ 是定域泛函。发散决定了单圈重整化作用量 $\Delta S = -\hbar\Gamma^{(1)}_{\text{div}}$ 中 counterterm(抵消项) 的形式，因此重整化作用量为

$$S_{1R} = S - \hbar\Gamma^{(1)}_{\text{div}}, \tag{99}$$

which is also a local functional.

它同样是定域泛函。

Let us substitute the expansion (98) in Eq. (95) and preserve terms up to the first order in $\hbar$. Thus, $\Gamma^{(1)}_{\text{div}}$ and $\Gamma^{(1)}_{\text{fin}}$ satisfy the equation

将展开式 (98) 代入方程 (95)，保留到 $\hbar$ 一阶项，可得 $\Gamma^{(1)}_{\text{div}}$ 和 $\Gamma^{(1)}_{\text{fin}}$ 满足方程

$$0 = (\Gamma, \Gamma) = (S, S) + 2\hbar\left(S, \Gamma^{(1)}_{\text{div}}\right) + 2\hbar\left(S, \Gamma^{(1)}_{\text{fin}}\right) + O\left(\hbar^2\right)$$

$$= 2\hbar\left(S, \Gamma^{(1)}_{\text{div}}\right) + 2\hbar\left(S, \Gamma^{(1)}_{\text{fin}}\right) + O\left(\hbar^2\right), \tag{100}$$

where we used the zero-order equation (37). In the first order in $\hbar$, we have a vanishing sum of the two terms, one of them is infinite, and hence it has to vanish independently on another one. Therefore

推导中我们用到了零阶方程 (37)。在 $\hbar$ 一阶下，我们得到两个项的和为零，其中一个项是无穷大，因此它必须独立于另一个项单独等于零，因此有

$$\left(S, \Gamma^{(1)}_{\text{div}}\right) = 0. \tag{101}$$

Let us consider

我们接下来考虑

$$(S_{1R}, S_{1R}) = (S, S) - 2\hbar\left(S, \Gamma^{(1)}_{\text{div}}\right) + \hbar^2\left(\Gamma^{(1)}_{\text{div}}, \Gamma^{(1)}_{\text{div}}\right). \tag{102}$$

Taking into account (37) and (101), we find the relation

结合 (37) 和 (101)，我们得到关系

$$(S_{1R}, S_{1R}) = \hbar^2 E_2, \tag{103}$$

where E_2 is the known functional. Thus, we have shown that S_{1R} satisfies the classical master equation (37) up to the terms of order $\hbar^2$:

其中 E_2 是已知泛函。由此我们证明了 S_{1R} 满足经典主方程 (37)，误差不超过 $\hbar^2$ 阶项:

$$E_2 = \left(\Gamma^{(1)}_{\text{div}}, \Gamma^{(1)}_{\text{div}}\right). \tag{104}$$

Let us now include one-loop corrections to S . The one-loop effective action Γ_{1R} can be constructed by adding a local counterterm to the $\mathcal{O}(\hbar)$ part of Eq. (98). As usual, the counterterm has the divergent part which cancels the divergence of $\Gamma^{(1)}_{\text{div}}$, and the remaining contribution is finite and typically depends on the renormalization parameter μ . The dependence on μ always repeats the form of divergence which, as we have seen above, does not violate the master equation. Thus, we can simply use (99) and assume that Γ_{1R} is constructed by following the procedure of quantization described above, with S replaced by S_{1R} .

现在我们引入对 S 的单圈修正。单圈有效作用量 Γ_{1R} 可以通过在式 (98) 的 $\mathcal{O}(\hbar)$ 部分添加定域抵消项构造得到。和通常情况一样，抵消项的发散部分会抵消 $\Gamma^{(1)}_{\text{div}}$ 的发散，剩余贡献是有限的，且通常依赖于重整化参数 μ 。对 μ 的依赖总是和发散的形式一致，而正如前文所见，这不会破坏主方程。因此我们可以直接利用 (99)，认为 Γ_{1R} 是按照前文所述的量子化流程构造的，只需将 S 替换为 S_{1R} 即可。

Being constructed in this way, the functional Γ_{1R} is finite in the one-loop approximation and satisfies the equation

按此方式构造的泛函 Γ_{1R} 在单圈近似下有限，且满足方程

$$(\Gamma_{1R}, \Gamma_{1R}) = \hbar^2 E_2 + O\left(\hbar^3\right). \tag{105}$$

Now, it is time to consider the next level of the loop expansion. Consider the one-loop renormalized effective action, taking into account the $\mathcal{O}\left(\hbar^2\right)$ -terms:

现在我们该考虑圈展开的下一阶了。考虑计入 $\mathcal{O}\left(\hbar^2\right)$ 项后的单圈重整化有效作用量:

$$\Gamma_{1R} = S + \hbar\Gamma^{(1)}_{\text{fin}} + \hbar^2\left[\Gamma^{(2)}_{1,\,\text{div}} + \Gamma^{(2)}_{1,\,\text{fin}}\right] + O\left(\hbar^3\right). \tag{106}$$

Here, $\Gamma^{(2)}_{1,\,\text{div}}$ and $\Gamma^{(2)}_{1,\,\text{fin}}$ are divergent and finite $\mathcal{O}\left(\hbar^2\right)$ parts of the two-loop effective action constructed on the basis of S_{1R} instead of S . The divergent part $\Gamma^{(2)}_{1,\,\text{div}}$ of the two-loop approximation for Γ_{1R} determines the two-loop renormalization for S_{2R} , according to

此处，$\Gamma^{(2)}_{1,\,\text{div}}$ 和 $\Gamma^{(2)}_{1,\,\text{fin}}$ 分别是基于 S_{1R} 而非 S 构造的双圈有效作用量的发散部分与有限部分 $\mathcal{O}\left(\hbar^2\right)$。双圈近似下 Γ_{1R} 的发散部分 $\Gamma^{(2)}_{1,\,\text{div}}$ 按下式确定 S_{2R} 的双圈重整化:

$$S_{2R} = S_{1R} - \hbar^2\Gamma^{(2)}_{1,div} \tag{107}$$

and satisfies the equation

且满足方程

$$\left(S, \Gamma_{1,\,\mathrm{div}}^{(2)}\right) = E_2.$$

As the next step, let us consider

下一步，我们来考虑

$$(S_{2R}, S_{2R}) = \hbar^3 E_3 + O\left(\hbar^4\right). \tag{108}$$

We have found that S_{2R} satisfies the master equations up to the terms $\hbar^3 E_3$, where

我们发现 S_{2R} 满足主方程，精度可达 $\hbar^3 E_3$ 项，其中

$$E_3 = 2\left(\Gamma_{\mathrm{div}}^{(1)}, \Gamma_{1,\,\mathrm{div}}^{(2)}\right), \tag{109}$$

The effective action Γ_{2R} is generated by replacing S_{2R} into functional integral instead of S . Therefore, Γ_{2R} is automatically finite in the two-loop approximation,

有效作用量 Γ_{2R} 通过将泛函积分中的 S 替换为 S_{2R} 生成。因此 Γ_{2R} 在双圈近似下自动有限,

$$\Gamma_{2R} = S + \hbar\Gamma_{\mathrm{fin}}^{(1)} + \hbar^2\Gamma_{1,\,\mathrm{fin}}^{(2)} + \hbar^3\left[\Gamma_{2,\,\mathrm{div}}^{(3)} + \Gamma_{2,\,\mathrm{fin}}^{(3)}\right] + O\left(\hbar^4\right)$$

and satisfies the equation

且满足方程

$$(\Gamma_{2R}, \Gamma_{2R}) = \hbar^3 E_3 + O\left(\hbar^4\right). \tag{110}$$

Furthermore, by applying the induction method, we find that the totally renormalized action S_R is given by the expression

进一步，通过归纳法可得，完全重整化作用量 S_R 由下式给出:

$$S_R = S - \sum_{n=1}^{\infty} \hbar^n \Gamma_{n-1,\,\mathrm{div}}^{(n)} \tag{111}$$

We assume that $\Gamma_{n-1,\,\mathrm{div}}^{(n)}$ and $\Gamma_{n-1,\,\mathrm{fin}}^{(n)}$ are the divergent and finite parts of the n -loop approximation for the effective action, which is already finite in $(n-1)$ - loop approximation, since it is constructed on the basis of the action $S_{(n-1)R}$.

我们假设 $\Gamma_{n-1,\,\mathrm{div}}^{(n)}$ 和 $\Gamma_{n-1,\,\mathrm{fin}}^{(n)}$ 分别是有效作用量 n 圈近似的发散部分与有限部分，该有效作用量在 $(n-1)$ 圈近似下已经有限，因为它是基于作用量 $S_{(n-1)R}$ 构造的。

The action (111) is a local functional and exactly satisfies the classical master equation

式 (111) 的作用量是定域泛函，精确满足经典主方程

$$(S_R, S_R) = 0. \tag{112}$$

We have shown that this relation holds in all orders of the loop expansion. This means that the BRST symmetry is preserved in the renormalized action S_R . Let us note that this corresponds exactly to the BRST cohomology on local functionals with ghost number zero [62, 63] .

我们已经证明该关系在圈展开的所有阶都成立，这意味着 BRST 对称性在重整化作用量 S_R 中得到保留。请注意，这恰好对应鬼数为零的定域泛函上的 BRST 上同调 [62, 63] 。

The renormalized effective action Γ_R is finite in each order of the loop expansion in the powers of $\hbar$:

重整化有效作用量 Γ_R 在以 $\hbar$ 幂次展开的圈展开各阶中都有限:

$$\Gamma_R = S + \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}_{n-1,\,\text{fin}}\,, \tag{113}$$

and satisfies the gravitational analog of the Slavnov-Taylor identities [8, 15, 16] in Yang-Mills theory (see also [1] for the pedagogical introduction), Thus, the renormalized action S_R and the effective action Γ_R satisfy the classical master equation and the gravitational version of Ward (Slavnov-Taylor) identity, respectively.

且满足杨-米尔斯理论中斯拉诺夫-泰勒恒等式 [8, 15, 16] 的引力类比 (入门介绍参见文献 [1])。因此，重整化作用量 S_R 满足经典主方程，有效作用量 Γ_R 满足沃德 (斯拉诺夫-泰勒) 恒等式的引力形式，二者分别成立。

Gauge Invariance of Renormalized Background Effective Action

重整化背景有效作用量的规范不变性

The next part of our consideration will be to generalize the transformation relations (82) and (94) for the renormalized functionals of classical S_R and effective Γ_R actions. As before, we consider the transformations perturbatively, starting from the lowest orders. In the one-loop approximation, from (94), it follows

我们接下来的研究将推广经典 S_R 作用量和有效 Γ_R 作用量的重整化泛函的变换关系 (82) 和 (94)。和之前一样，我们从最低阶开始，微扰地研究这些变换。在单圈近似下，由 (94) 可得

$$\delta_\omega \Gamma\left(\Phi, \Phi^*, \bar{g}\right) = \Phi^*_i \delta_\omega R^i\left(\Phi, \bar{g}\right) + \hbar \Phi^*_i \delta_\omega \bar{R}^{i(1)}_{\text{div}}\left(\Phi, \Phi^*, \bar{g}\right)$$

$$+\hbar \Phi^*_i \delta_\omega \bar{R}^{i(1)}_{\text{fin}}\left(\Phi, \Phi^*, \bar{g}\right) + O\left(\hbar^2\right), \tag{114}$$

where we used the condensed notations (91). In the last expression,

此处我们使用了缩记符号 (91)。在上一表达式中,

$$\delta_\omega \bar{R}^{i(1)}_{\text{div}}(\Phi,\Phi^*,\bar{g}) \quad \text{and} \quad \delta_\omega \bar{R}^{i(1)}_{\text{fin}}(\Phi,\Phi^*,\bar{g})$$

are divergent and finite parts of the one-loop approximation for the gauge transformations $\delta_\omega \bar{R}^i(\Phi,\Phi^*,\bar{g})$, correspondingly. On the other hand, from (98), we have

分别是规范变换 $\delta_\omega \bar{R}^i(\Phi,\Phi^*,\bar{g})$ 单圈近似的发散部分和有限部分。另一方面，由 (98) 我们得到

$$\delta_\omega \Gamma(\Phi,\Phi^*,\bar{g}) = \delta_\omega S(\Phi,\Phi^*,\bar{g}) + \hbar\delta_\omega \Gamma^{(1)}_{\text{div}} + \hbar\delta_\omega \Gamma^{(1)}_{\text{fin}} + O\left(\hbar^2\right). \tag{115}$$

The comparison of the two transformations (114) and (115) tells us that

对比两个变换式 (114) 和 (115)，我们可得

$$\delta_\omega \Gamma^{(1)}_{\text{div}} = \Phi^*_i \delta_\omega \bar{R}^{i(1)}_{\text{div}}(\Phi,\Phi^*,\bar{g}), \tag{116}$$

$$\delta_\omega \Gamma^{(1)}_{\text{fin}} = \Phi^*_i \delta_\omega \bar{R}^{i(1)}_{\text{fin}}(\Phi,\Phi^*,\bar{g}). \tag{117}$$

From Eq. (116) and the definition (99), it follows that the one-loop renormalized action $S_{1R} = S_{1R}(\Phi,\Phi^*,\bar{g})$ transforms according to

由式 (116) 和定义 (99) 可知，单圈重整化作用量 $S_{1R} = S_{1R}(\Phi,\Phi^*,\bar{g})$ 按照如下规则变换

$$\delta_\omega S_{1R} = \Phi^*_i \delta_\omega R^{i(1)}_R, \tag{118}$$

where

其中

$$R^{i(1)}_R = R^{i(1)}_R(\Phi,\Phi^*,\bar{g}) = \delta_\omega R^i(\Phi,\bar{g}) - \hbar\delta_\omega \bar{R}^{i(1)}_{\text{div}}(\Phi,\Phi^*,\bar{g}). \tag{119}$$

The last relations mean that the action S_{1R} is invariant under the background gauge transformations with one-loop deformed gauge generators $R^{i(1)}_R$ (119) on the hypersurface $\Phi^* = 0$. Furthermore, due to Eq. (118), functional Γ_{1R} obeys the transformation rule

上述关系表明，在超曲面 $\Phi^* = 0$ 上，作用量 S_{1R} 在带有单圈形变规范生成元 $R^{i(1)}_R$ (119) 的背景规范变换下不变。此外，由式 (118) 可知，泛函 Γ_{1R} 满足如下变换规则

$$\delta_\omega \Gamma_{1R} = \Phi^*_i \delta_\omega R^i + \hbar\Phi^*_i \delta_\omega \bar{R}^{i(1)}_{\text{fin}}$$

$$+\hbar^2\left[\Phi^*_i \delta_\omega \bar{R}^{i(2)}_{1,\text{div}} + \Phi^*_i \delta_\omega \bar{R}^{i(2)}_{1,\text{fin}}\right] + O\left(\hbar^3\right), \tag{120}$$

where $\delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{div}} = \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{div}}(\Phi, \Phi^*, \bar{g})$ and $\delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{fin}} = \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{fin}}(\Phi, \Phi^*, \bar{g})$ are related to $\Gamma^{(2)}_{1,\,\mathrm{div}}$ and $\Gamma^{(2)}_{1,\,\mathrm{fin}}$ in Eq. (106) as

其中 $\delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{div}} = \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{div}}(\Phi, \Phi^*, \bar{g})$ 和 $\delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{fin}} = \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{fin}}(\Phi, \Phi^*, \bar{g})$ 与式 (106) 中的 $\Gamma^{(2)}_{1,\,\mathrm{div}}$ 和 $\Gamma^{(2)}_{1,\,\mathrm{fin}}$ 满足关系

$$\delta_\omega \Gamma^{(2)}_{1,\,\mathrm{div}} = \Phi^*_i \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{div}}\,,\ \delta_\omega \Gamma^{(2)}_{1,\,\mathrm{fin}} = \Phi^*_i \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{fin}}\,. \tag{121}$$

Therefore, the functional Γ_{1R} is finite in the one-loop approximation and is invariant under the background gauge transformations up to the second order in $\hbar$ on the hypersurface $\Phi^* = 0$.

因此，在超曲面 $\Phi^* = 0$ 上，泛函 Γ_{1R} 在单圈近似下有限，且在背景规范变换下 $\hbar$ 二阶范围内不变。

Applying the induction method, one can show that the renormalized functionals S_R and Γ_R satisfy the properties.

利用归纳法可以证明，重整化泛函 S_R 和 Γ_R 满足上述性质。

$$\delta_\omega S_R = \Phi^*_i \delta_\omega R^i_R,\ \delta_\omega \Gamma_R = \Phi^*_i \delta_\omega \bar{R}^i_R, \tag{122}$$

where

其中

$$\delta_\omega R^i_R = \delta_\omega R^i - \hbar \delta_\omega \bar{R}^{i(1)}_{\mathrm{div}} - \hbar^2 \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{div}} - \cdots, \tag{123}$$

$$\delta_\omega \bar{R}^i_R = \delta_\omega R^i + \hbar \delta_\omega \bar{R}^{i(1)}_{\mathrm{fin}} + \hbar^2 \delta_\omega \bar{R}^{i(2)}_{1,\,\mathrm{fin}} + \cdots. \tag{124}$$

It is important that $\delta_\omega \bar{R}^i_R$, defined in (124), are finite. (We note that these statements are very close to the results concerning preservation of global symmetries of initial classical action at quantum level when the effective action of theory under consideration is invariant under deformed global transformations of all fields [64].)

(124) 定义的 $\delta_\omega \bar{R}^i_R$ 是有限的，这一点十分重要。(我们注意到，这些结论与下述结论非常接近: 当所研究理论的有效作用量在所有场的形变整体变换下不变时，初始经典作用量的整体对称性在量子层面得以保持 [64]。)

As it was discussed above, the UV divergences in any models of QG are local, even for the nonlocal QG theories [21, 22, 61] . As a consequence, the quantities $\delta_\omega R^i_R$, defined in (123), are local while in both local and nonlocal models of QG, including the proper transformations δ_ω . Then, an important consequence of the results (122) is that the renormalized functionals $S_R(\Phi, \bar{g}) = S_R(\Phi, \Phi^* = 0, \bar{g})$ and $\Gamma_R(\Phi, \bar{g}) = \Gamma_R(\Phi, \Phi^* = 0, \bar{g})$ obey the symmetry

正如前文所讨论的，任何量子引力 (QG) 模型中的紫外发散都是定域的，即便是非局域量子引力理论 [21, 22, 61] 也是如此。因此，(123) 定义的量 $\delta_\omega R_R^i$ 是定域的，这在包括正则变换 δ_ω 在内的定域和非局域量子引力模型中都成立。那么，结果 (122) 的一个重要结论是，重整化泛函 $S_R(\Phi,\bar{g}) = S_R(\Phi,\Phi^* = 0,\bar{g})$ 和 $\Gamma_R(\Phi,\bar{g}) = \Gamma_R(\Phi,\Phi^* = 0,\bar{g})$ 满足对称性

$$\delta_\omega S_R(\Phi,\bar{g}) = 0,\ \delta_\omega \Gamma_R(\Phi,\bar{g}) = 0. \tag{125}$$

These are the same transformations as we met for the non-renormalized functionals $S(\Phi,\bar{g}) = S_{FP}(\Phi,\bar{g})$ and $\Gamma(\Phi,\bar{g})$ in (67) and (79), respectively. Therefore, from (125), it follows the invariance property of the renormalized background functionals $S_R(\bar{g}) = S(\Phi = 0,\bar{g})$ and $\Gamma(\bar{g}) = \Gamma(\Phi = 0,\bar{g})$ under general coordinate transformations of external background metric $\bar{g}_{\mu\nu}$:

这些变换与我们之前分别在 (67) 和 (79) 中遇到的非正规化泛函 $S(\Phi,\bar{g}) = S_{FP}(\Phi,\bar{g})$ 和 $\Gamma(\Phi,\bar{g})$ 的变换完全相同。因此，由 (125) 可推得，在外背景度规 $\bar{g}_{\mu\nu}$ 的广义坐标变换下，重整化背景泛函 $S_R(\bar{g}) = S(\Phi = 0,\bar{g})$ 和 $\Gamma(\bar{g}) = \Gamma(\Phi = 0,\bar{g})$ 满足不变性:

$$\delta_\omega^{(c)} S_R(\bar{g}) = 0,\ \delta_\omega^{(c)} \Gamma_R(\bar{g}) = 0. \tag{126}$$

It is easy to see that these invariances repeat exactly the symmetry properties of initial action $S_0(\bar{g})$ and $\Gamma(\bar{g})$ in (68).

不难看出，这些不变性完全重现了 (68) 中初始作用量 $S_0(\bar{g})$ 和 $\Gamma(\bar{g})$ 的对称性。

A Short Historical Review and More Special Notes

简短历史回顾与补充说明

In order to understand better the relevance of the results described above, let us start by presenting a short historical review of the subject. The first proof of the gauge invariant renormalizability in QG was given by Stelle in the famous 1977 paper [17]. Despite the considerations in this paper being done for the simplest renormalizable model of quantum gravity with four derivatives, most of the analysis is sufficiently general and can be applied, also, to other covariant models of QG. After that, there were further publications devoted to the invariant renormalizability in QG [34,35,37,38], where the main conclusion about the diffeomorphism invariant renormalizability has been confirmed with different degrees of generality and using formally different methods, regardless if all of the proofs are based on the BRST symmetry.

为了更好地理解上文所述结果的意义，我们首先对该研究方向做一简短历史回顾。量子引力 (QG) 中规范不变可重整性的首个证明由 Stelle 在 1977 年的著名论文 [17] 中给出。尽管该文针对的是带四阶导数的最简可重整量子引力模型，但多数分析足够具有一般性，也可适用于量子引力的其他协变模型。此后，有多篇进一步研究量子引力中不变可重整性的工作 [34,35,37,38]，尽管所有证明都基于 BRST 对称性，但这些工作利用形式不同的方法，在不同的普遍程度上确认了微分同胚不变可重整性的核心结论。

Since QG is a particular example of gauge theory, the program of exploring invariant renormalizability cannot be separated from the works done in the framework Yang-Mills theories. The most significant achievement in this respect was the demonstration of BRST invariant renormalizability in the theories which may be not renormalizable by power counting. The gauge invariant renormalizability, independent on the power counting, is especially important for the effective QFT [1] and, in particular, for effective approach to QG. The interested reader can consult a specially devoted to effective QG Section of the present Handbook for further details. In 1982, it was formulated the first proof for the general gauge theories [65], based on the Batalin-Vilkovisky formalism [49,50]. The approach employed in [65] assumed the regularization procedure respecting the gauge invariance of initial classical action and providing zero volume divergences, i.e., with the condition $\delta(0) = 0$ satisfied. The proof is valid for any boundary condition related to an initial gauge invariant action and for arbitrary choice of gauge-fixing functions. Furthermore, the renormalization procedure of [65] can be described in terms of anticanonical transformations (for recent developments, see [66, 67]) which are defined as transformations preserving the antibracket, using the terminology of the standard review paper [68].

由于量子引力是规范理论的一个特例，研究不变可重整性的工作离不开杨-米尔斯理论框架下的已有成果。该方向最重要的进展是证明了按幂次计数可能不可重整的理论依然具有 BRST 不变可重整性。不依赖幂次计数的规范不变可重整性对有效量子场论 [1]，尤其是量子引力的有效方法尤为重要。感兴趣的读者可以查阅本手册中专门介绍有效量子引力的章节获取更多细节。1982 年，基于巴塔林-维尔可维斯基形式 [49,50]，研究者给出了一般规范理论的首个证明 [65]。文献 [65] 采用的方案要求正则化过程满足初始经典作用量的规范不变性，且给出零体积发散，也就是满足条件 $\delta(0) = 0$ 。该证明对与初始规范不变作用量相关的任意边界条件，以及任意规范固定函数选择都成立。此外，文献 [65] 的重整化过程可以用反典则变换描述 (最新进展参见 [66,67])，按照经典综述文献 [68] 的术语，这类变换定义为保持反括号的变换。

An alternative, albeit very close, approach to prove the BRST invariant renormalization of general gauge theories [69], is based on the use of cohomologies of nilpotent BRST operator associated with the adjoint operation of the antibracket of the action S with an arbitrary functional $F, \hat{s}F = (S, F)$ [62, 63] . The detailed description of this approach can be found in [68] and in the chapter 17.3 of Vol. II of the book [1]. The disadvantage of this approach is that it does not directly extend to the background field method.

另一种证明一般规范理论 BRST 不变重整化的方法 [69] 与之非常接近，该方法利用幂零 BRST 算子的上同调，该算子对应作用量 S 和任意泛函 $F, \hat{s}F = (S, F)$ [62, 63] 的反括号伴随运算。该方法的详细描述可以在文献 [68] 以及著作 [1] 第二卷第 17.3 章找到。该方法的缺点是不能直接推广到背景场方法。

On the other hand, the background field formalism [7, 41, 42] represents a powerful approach to study quantum properties of gauge theories, allowing to keep the gauge invariance (general covariance, for QG), at all stages of the loop expansion, including in practical calculations. From the viewpoint of the quantization of gauge systems, the background field method corresponds to the special choice of a boundary condition and to the special choice of gauge-fixing functions. However, since the background field method requires the presence of an ”external” field in the course of the Lagrangian quantization, this formalism should be considered as a very special case which requires special care. Indeed, this special case attracted a great deal of attention recently [37, 43-46].

另一方面，背景场形式体系 [7, 41, 42] 是研究规范理论量子性质的有力方法，它可以在圈展开的所有阶段 (包括实际计算中) 保持规范不变性 (对量子引力而言是广义协变性)。从规范系统量子化的角度看，背景场方法对应边界条件和规范固定函数的特殊选择。但由于背景场方法要求拉格朗日量子化过程中存在一个“外”场，因此该形式体系是需要特殊处理的特例，而该特例近年来也确实得到了大量关注 [37, 43-46]。

A few more special notes on the structure of the gauge algebra underlying a given classical system are in order. Already the quantization of gauge systems using the Faddeev-Popov approach [6] should be applied to the theories when the gauge algebra is associated with a Lie group, as, otherwise, the quantization may give wrong results. The reason is that, in these more complicated cases, the gauge algebra may be reducible/open [70, 71] or structure functions may depend on the fields of the initial gauge theory, as it happens in supergravity [72-75]. In these cases, the symmetry transformations leave the action invariant, but do not form a gauge group. Thus, the quantization of gauge theories require taking into account not only the invariance of the action but also such important aspects as open/closed algebras, the presence of reducible generators, and so on. The quantization of these complicated theories is possible using different types of ghosts, antighosts, ghosts for ghosts, Nielsen-Kallosh ghosts, etc. [71, 74-80] (for recent applications to quantization of reducible gauge theories, see [81-83]). The most general (and the unique completely self-consistent) approach to the problem of covariant quantization, which summarized all these approaches, was proposed by Batalin and Vilkovisky [49, 50]. The Batalin-Vilkovisky formalism gives the rules of quantization of the general gauge theories which may be characterized by open/reducible gauge algebras, with structure functions depending on the fields of the initial action. The Batalin-Vilkovisky formalism is not only a powerful quantization method, as it also enables us to explore various complicated subjects and issues in gauge theories [65, 66, 69, 84-88]. And among these important applications, the renormalization of the quantum gauge theories of different types is one of the main problems. Coming back to gravity, in the next sections, we will see that QG belongs to the theories of the Yang-Mills type, i.e., it admits the traditional Faddeev-Popov approach and enables one to prove the gauge invariant renormalizability [17] (see also [35]). However, using the Batalin-Vilkovisky formalism, we can make this proof more elegant and simple, as we shall see in the rest of this review.

有必要对给定经典系统背后的规范代数结构补充若干特殊说明。当规范代数对应李群时，才能使用法捷耶夫-波波夫方法 [6] 对规范系统进行量子化，否则量子化可能得到错误结果。原因在于，在这些更复杂的情形中，规范代数可能是可约的/开代数 [70, 71] ，或者结构函数依赖于初始规范理论的场，超引力 [72-75] 中就存在这种情况。在这些情形中，对称变换保持作用量不变，但不构成规范群。因此，规范理论的量子化不仅需要考虑作用量的不变性，还需要考虑开代数/闭代数、存在可约生成元等重要方面。这类复杂理论可以通过不同类型的鬼、反鬼、鬼的鬼、尼尔森-卡勒什鬼等实现量子化 [71, 74-80](关于可约规范理论量子化的最新应用，参见 [81-83])。巴塔林-维尔可维斯基提出了解决协变量子化问题最通用 (也是唯一一个完全自洽) 的方法，整合了上述所有研究方案 [49, 50]。巴塔林-维尔可维斯基形式给出了广义规范理论的量子化规则，这类规范理论可以具有开/可约规范代数，且结构函数依赖于初始作用量的场。巴塔林-维尔可维斯基形式不仅是一种强大的量子化方法，还能帮助我们研究规范理论中各种复杂的课题与问题 [65, 66, 69, 84-88]。在这些重要应用中，不同类型量子规范理论的重整化是核心问题之一。回到引力的话题，我们将在后续章节看到，量子引力属于杨-米尔斯型理论，也就是说它适用传统的法捷耶夫-波波夫方法，还可以证明它是规范不变可重整的 [17](另见 [35])。但正如我们将在本综述余下部分看到的，借助巴塔林-维尔可维斯基形式，我们可以让这个证明更简洁优雅。

On the Gauge Fixing in the Higher Derivative Models

高阶导数模型中的规范固定

Equations (122) show that with the antifields switched off, i.e., with $\Phi^* = 0$, both the renormalized classical action S_R and effective action Γ_R are gauge invariant quantities. In particular, this means that if we restrict the consideration by the standard non-extended generating functional of the Green functions, that means, without introducing sources for the ghosts $C, \bar{C}$ and for the auxiliary field B , the effective action will be metric-dependent and generally covariant functional. This statement concerns both divergent and finite parts of renormalized effective action. In what follows, we discuss how the gauge invariant renormalizability can be applied to the renormalization of the models of QG. The rest of this section partially repeats some part of - Chap. 8, "The Background Information About Perturbative Quantum Gravity", but we shall discuss the subject from a different perspective.

式 (122) 表明，当反场关闭即满足 $\Phi^* = 0$ 时，重整化经典作用量 S_R 和有效作用量 Γ_R 均为规范不变量。具体而言，这意味着如果我们仅考虑标准的非延拓格林函数生成泛函，即不引入鬼场 $C, \bar{C}$ 和辅助场 B 的源，那么有效作用量将是一个度规相关的广义协变泛函。该结论对重整化有效作用量的发散部分和有限部分均成立。下文我们将讨论规范不变可重整性如何应用于量子引力模型的重整化。本节剩余内容部分重复了第 8 章“微扰量子引力背景知识”的部分内容，但我们会从不同视角展开讨论。

The use of the power counting arguments in QG models may be especially simple if the following two conditions are satisfied:

若满足以下两个条件，在量子引力模型中使用幂计数论证会格外简单:

(i) The propagator of the gravitational field should be homogeneous in the powers of momenta. This means, in particular, that the free equations for different modes of the gravitational field (tensor, vector, and scalar) are of the same order in derivatives after the Faddeev-Popov procedure.

(i) 引力场传播子应对动量幂次齐次。这意味着，在法捷耶夫-波波夫 procedure 之后，引力场不同模式 (张量、矢量、标量) 的自由运动方程对导数的阶数相同。

(ii) The propagator of FP ghosts should have the same powers of momenta as all modes of the gravitational field.

(ii) FP 鬼场的传播子应与引力场所有模式具有相同的动量幂次。

As we explained in detail in the previous chapter, these conditions are fulfilled if the gauge-fixing term and the ghost action have the form

正如我们在上一章中详细解释的，当规范固定项和鬼场作用量取如下形式时，这两个条件成立:

$$S_{gf} = \int d^4x \sqrt{-g}\,\chi^\alpha Y_{\alpha\beta} \chi^\beta, \tag{127}$$

$$S_{gh} = \int d^4x\sqrt{-g}\bar{C}^{\alpha}Y_{\alpha\beta}M^{\beta}_{\lambda}C^{\lambda}, \tag{128}$$

where, according to (17) and (18),

其中，根据式 (17) 和 (18),

$$M^{\beta}_{\lambda} = H^{\rho\sigma}_{\beta}(x, y; \bar{g}, h)\, R_{\rho\sigma\lambda}(y, z; \bar{g} + h)\,. \tag{129}$$

The choice of the weight operator $Y_{\alpha\beta}$ should be done in such a way that the total amount of derivatives in the expressions (127) and (128) be the same as in the action of the model of quantum gravity under consideration. For instance, in the QG based on general relativity, $Y_{\alpha\beta} = \theta g_{\alpha\beta}$, where θ is a constant gauge-fixing parameter. In case of the fourth-order gravity, one has to take [17, 89, 90]

权重算符 $Y_{\alpha\beta}$ 的选取应满足: 式 (127) 和 (128) 中导数的总次数与所研究量子引力模型作用量中的导数总次数相等。例如，在基于广义相对论的量子引力中，$Y_{\alpha\beta} = \theta g_{\alpha\beta}$ ，其中 θ 是常数规范固定参数。对于四阶引力，需取 [17, 89, 90]

$$Y_{\alpha\beta} = \theta_1\delta_{\alpha\beta}\Box + \theta_2\nabla_{\alpha}\nabla_{\beta} + \theta_3 R_{\alpha\beta} + \theta_4\delta_{\alpha\beta}R, \tag{130}$$

where $\theta_{1,2,3,4}$ are gauge-fixing constants. In the case of six-derivative superrenor-malizable gravity model [18], $\theta_{1,2,3,4}$ should be linear functions of d’Alembertian operator $\Box$, plus the possible linear in curvature tensor terms, for the eight-derivative QG the parameters $\theta_{1,2,3,4}$ become quadratic functions of $\Box$, etc.

其中 $\theta_{1,2,3,4}$ 是规范固定常数。对于六导数超可重整化引力模型 [18]，$\theta_{1,2,3,4}$ 应是达朗贝尔算符 $\Box$ 的线性函数，还可包含曲率张量的线性项；对于八导数量子引力，参数 $\theta_{1,2,3,4}$ 则变为 $\Box$ 的二次函数，依此类推。

An important question is how to incorporate the modified gauge-fixing and ghost actions (127) and (128) into the proof of gauge invariant renormalizability which we developed in section ”Gauge Invariant Renormalizability.” The simplest possibility in this direction is to remember that the effective action in the super-renormalizable QG theories with more than four derivatives does not depend on the gauge fixing [18]. Thus, the scheme based on the weight function (130) with $\theta_{1,2,3,4}$ linear functions of $\Box$ does not affect the loop corrections. This argument looks convincing, but let us present some extra details and refer to [38] for further discussions.

一个重要问题是，如何将修正后的规范固定和鬼场作用量 (127)、(128) 纳入我们在“规范不变可重整性”一节中给出的规范不变可重整性证明。该方向最简单的思路是注意到，导数阶数大于四的超可重整化量子引力理论中，有效作用量不依赖规范固定 [18]。因此，基于权重函数 (130) 且 $\theta_{1,2,3,4}$ 为 $\Box$ 线性函数的方案不会影响圈修正。该论证虽然可信，但我们仍将给出额外细节，并可参考 [38] 获取进一步讨论。

Consider $\chi_{\alpha} = \chi_{\alpha}(x; \bar{g}, h)$ being a standard gauge-fixing function used in previous sections. We can introduce the set of two differential operators, Y^{β}_{α} and $Y_{1\alpha\beta}$. These weight operators must have the structure of tensor fields of types $(1, 1)$ and $(0, 2)$, respectively, and cannot depend on the quantum metric $h_{\mu\nu}$:

设 $\chi_\alpha = \chi_\alpha(x;\bar{g},h)$ 为前几节使用的标准规范固定函数。我们引入两个微分算符集合 Y_α^β 和 $Y_{1\alpha\beta}$ 。这些权重算符需分别具有 $(1,1)$ 型和 $(0,2)$ 型张量场结构，且不依赖于量子度规 $h_{\mu\nu}$:

$$Y_\alpha^\beta(x,y) = Y_\alpha^\beta\left(x,y;\bar{g},\overline{\Box}\right) \quad \text{and} \quad Y_{1\alpha\beta}(x,y) = Y_{1\alpha\beta}\left(x,y;\bar{g},\overline{\Box}\right). \tag{131}$$

The next step is to modify the gauge-fixing functions χ_α , by the following rule:

下一步我们按如下规则修改规范固定函数 χ_α :

$$\chi_\alpha^{\text{mod}}(x;\bar{g},h,B)$$

$$= \int dy\left[Y_\alpha^\beta\left(x,y;\bar{g},\overline{\Box}\right)\chi_\beta(y;\bar{g},h) + \frac{1}{2}Y_{1\alpha\beta}\left(x,y;\bar{g},\overline{\Box}\right)B^\beta(y)\right] \tag{132}$$

and construct the corresponding gauge-fixing functional:

随后构造对应的规范固定泛函:

$$\Psi^{\text{mod}}(\phi,\bar{g}) = \int dx\sqrt{-\bar{g}}\bar{C}^\alpha(x)\,\chi_\alpha^{\text{mod}}(x;\bar{g},h,B). \tag{133}$$

According to what we previously learned, the transformation law of χ_α^{mod} coincides with the transformation rule of tensor fields of type $(0,1)$. Then the modified Faddeev-Popov action is constructed in the standard manner, using the generator of BRST transformations, $\hat{R}(\phi,\bar{g})$:

根据我们此前的结论，χ_α^{mod} 的变换规律与 $(0,1)$ 型张量场的变换规则一致。随后利用 BRST 变换的生成元 $\hat{R}(\phi,\bar{g})$ ，按标准方法构造出修正后的法捷耶夫-波波夫作用量:

$$S_{FP}^{\text{mod}}(\phi,\bar{g}) = S_0(\bar{g}+h) + \Psi^{\text{mod}}(\phi,\bar{g})\,\hat{R}(\phi,\bar{g}). \tag{134}$$

The explicit form of the second term in the right-hand side of (134) is

(134) 右侧第二项的显式形式为

$$\Psi^{\text{mod}}(\phi,\bar{g})\,\hat{R}(\phi,\bar{g}) = \int dxdydzdu\sqrt{-\bar{g}(x)}\bar{C}^\alpha(x)\,Y_\alpha^\beta\left(x,u;\bar{g},\overline{\Box}\right)$$

$$\times H_\beta^{\gamma\sigma}(u,y;\bar{g},h)\,R_{\gamma\sigma\rho}(y,z;\bar{g}+h)\,C^\rho(z)$$

$$+\int dxdy\sqrt{-\bar{g}(x)}\left[B^\alpha(x)\,Y_\alpha^\beta\left(x,y;\bar{g},\overline{\Box}\right)\chi_\beta(y;\bar{g},h)\right.$$

$$\left.+\frac{1}{2}B^\alpha(x)\,Y_{1\alpha\beta}\left(x,y;\bar{g},\overline{\Box}\right)B^\beta(y)\right].$$

The first term in the r.h.s. of the last formula is exactly (128) with (129). As the transformation rules for the terms in the Faddeev-Popov action depend only on the type of the tensor fields, all the main statements of the previous sections remain valid for the new choice of the gauge-fixing functions (132).

上式右侧第一项正是结合 (129) 后的 (128)。由于法捷耶夫-波波夫作用量中各项的变换规则仅由张量场的类型决定，对于规范固定函数 (132) 的新选取，前述各节所有核心结论仍然成立。

To deal only with the problem of the homogeneity of the propagator of the quantum metric $h_{\mu\upsilon}$, consider a special choice of the weight operator

为专门研究量子度规 $h_{\mu\nu}$ 传播子的齐次性问题，我们考虑权重算符的一种特殊选取

$$
Y_{1\alpha\beta}(x,y)=\bar{g}_{\alpha\gamma}(x)\left(Y^{-1}\right)^{\gamma}_{\beta}(x,y),
$$

$$
\text{where}\int dzY^{\gamma}_{\alpha}(x,z)\left(Y^{-1}\right)^{\beta}_{\gamma}(z,y)=\delta^{\beta}_{\alpha}\delta(x-y)\ ,
$$

$$
Y^{\beta}_{\alpha}(x,y)=Y^{\beta}_{\alpha}\left(x;\bar{g},\overline{\Box}\right)\delta(x-y). \tag{135}
$$

Integrating over the fields B^{α} in the functional integral defines the generating functional of Green functions in terms of integration over fields $\bar{C}^{\alpha},C^{\alpha}$, and $h_{\mu\nu}$. As a result, we obtain the functional determinant that is equal to

在泛函积分中对场 B^{α} 积分后，格林函数的生成泛函可表示为对场 $\bar{C}^{\alpha},C^{\alpha}$ 和 $h_{\mu\nu}$ 积分的形式，最终得到的泛函行列式等于

$$
\left[\mathrm{Det}\,Y^{\beta}_{\alpha}(x,y)\right]^{1/2}, \tag{136}
$$

independent on the variables of integration. It is worth noting that the factor (136) is well-known in higher derivative QG models [18,28,90].

它与积分变量无关。值得注意的是，因子 (136) 在高阶导数量子引力模型中已是熟知结论 [18,28,90]。

After all, to introduce a nontrivial weight operator, we need to replace

总而言之，要引入非平凡权重算符，我们需要将

$$
\Psi^{\mathrm{mod}}(\phi,\bar{g})\,\hat{R}(\phi,\bar{g})+\int dx\sqrt{-\bar{g}(x)}J^{(B)}_{\alpha}(x)\,B^{\alpha}(x) \tag{137}
$$

by the more complicated expression

替换为更复杂的表达式

$$
\int dxdydz\sqrt{-\bar{g}(x)}\bar{C}^{\alpha}(x)\,Y^{\beta}_{\alpha}\left(x;\bar{g},\overline{\Box}\right)H^{\gamma\sigma}_{\beta}(x,y;\bar{g},h)\,R_{\gamma\sigma\rho}(y,z;\bar{g}+h)\,C^{\rho}(z)
$$

$$-\frac{1}{2}\int dx\sqrt{-\bar{g}(x)}J^{(B)\alpha}(x)\,Y_{\alpha}^{\beta}\left(x;\bar{g},\overline{\Box}\right)J_{\beta}^{(B)}(x)$$

$$-\int dx\sqrt{-\bar{g}(x)}J_{\alpha}^{(B)}(x)\,\chi^{\alpha}(x;\bar{g},h)$$

$$-\frac{1}{2}\int dx\sqrt{-\bar{g}(x)}\chi^{\alpha}(x;\bar{g},h)\,Y_{\alpha}^{\beta}\left(x;\bar{g},\overline{\Box}\right)\chi_{\beta}(x;\bar{g},h)\,, \tag{138}$$

where the notations

其中使用了记号

$$\chi^{\alpha}(x;\bar{g},h)=\bar{g}^{\alpha\beta}(x)\,\chi_{\beta}(x;\bar{g},h)\,,$$

$$J^{(B)\alpha}(x)=\bar{g}^{\alpha\beta}(x)\,J_{\beta}^{(B)}(x) \tag{139}$$

are used. The second term in the expression (138) is exactly what is needed for the homogeneity condition (127). The total action appearing after integration over fields B^{α} is the sum of the action $S_0(\bar{g}+h)$ plus the first and the fourth terms in (138). This total action is invariant under the BRST transformations; the last are now recast in the form

。表达式 (138) 中的第二项恰好满足齐次性条件 (127) 的要求。对场 B^{α} 积分后得到的总作用量，是作用量 $S_0(\bar{g}+h)$ 加上 (138) 中第一项和第四项的和。该总作用量在 BRST 变换下不变，此时 BRST 变换可改写为如下形式

$$\delta_B h_{\mu\nu}(x)=\int dyR_{\mu\nu\alpha}(x,y;\bar{g}+h)\,C^{\alpha}(y)\,\mu,$$

$$\delta_B C^{\alpha}(x)=-C^{\sigma}(x)\,\partial_{\sigma}C^{\alpha}(x)\,\mu, \tag{140}$$

$$\delta_B\bar{C}^{\alpha}(x)=-\chi^{\alpha}(x;\bar{g},h)\,\mu.$$

In this form, the BRST symmetry also enables us to use all the basic properties to explore the renormalization of quantum theory, as one can verify by inspecting the considerations in sections ”Quantum Gravity in the Background Field Formalism” and ”Gauge Invariant Renormalizability” (see also [17]). However, there is a price to pay for the possibility to work with the functional integral over the smaller number of integration variables. One can note that the nilpotency property of the BRST transformations is lost in the new version of BRST transformations (140). On another hand, in the context of gauge invariant renormalizability, the nilpotency of the BRST transformation does not play a critical role. For this reason, we can freely switch between the two versions of BRST, according to our convenience.

在这种形式下，BRST 对称性仍允许我们利用所有基本性质研究量子理论的重整化，这一点可以通过考察“背景场形式下的量子引力”和“规范不变可重整性”两节的讨论验证 (参见文献 [17])。不过，在更少积分变量的泛函积分中工作是有代价的: 可以看到，在新版 BRST 变换 (140) 中，BRST 变换的幂零性丢失了。另一方面，在规范不变可重整性的框架下，BRST 变换的幂零性并不起关键作用。因此，我们可以根据需要随意在两种 BRST 形式间切换。

As the problem of homogeneity and introduction of (127) and (128) has been solved, we are in a position to review the power counting and classify the models of quantum gravity, as it was explained in detail in Chap. 8, ”The Background Information About Perturbative Quantum Gravity” (see also [38] and [91]). Thus, the important understanding of the main aspects of renormalization and effective approaches in perturbative QG gains a reliable basis after we get a mathematically solid proof of the gauge invariant renormalizability of the classically covariant models of gravity.

既然齐次性问题以及 (127)、(128) 的引入问题都已解决，我们就可以像第 8 章“微扰量子引力背景知识”中详细解释的那样，重新进行幂次计数并对量子引力模型分类 (也参见文献 [38] 和 [91])。在我们得到经典协变引力模型规范不变可重整性的严格数学证明后，对微扰量子引力中重整化和有效方法核心方面的重要认识就有了可靠基础。

Conclusions

结论

The general proof of diffeomorphism invariant renormalization in QG, independent of the renormalizability by power counting, is relevant for several important reasons. In particular, such a proof provides a solid basis for the low-energy effective approach in QG, which means making practical calculations in the low-energy sector of the theory, even in non-renormalizable models. In this case, it is necessary to be sure that the UV divergences are subtracted by local covariant counterterms that do not affect the physics in the IR [1].

量子引力 (QG) 中不依赖幂次计数可重整性的微分同胚不变重整化的通用证明之所以重要，有几个关键原因。尤其是该证明为量子引力的低能有效方法提供了坚实基础——这意味着哪怕在非可重整模型中，也能在理论的低能区开展实用计算。在这种情况下，我们必须确认紫外发散可由不影响红外区物理的局域协算 counterterm 抵消 [1]。

The main advantages of the approach of [38], which we reproduced and reviewed here, are related to the compact description of the variation of extended effective action under the gauge transformations of all fields used in the background field formalism. The derived form of these variations can be applied to an arbitrary QG theory, respecting the diffeomorphism invariance. The variation shows an invariance of the effective action when the antifields (sources for the BRST generators) are switched off.

本文重现并综述的文献 [38] 方法的核心优势，在于它对背景场形式体系中所有场的规范变换下，扩展有效作用量变化给出了紧致描述。推导出的这种变化形式可应用于任何满足微分同胚不变性的量子引力理论。当反场 (BRST 生成元的源) 关闭后，该变化表明有效作用量满足不变性。

After switching off the mean field of quantum metric, Faddeev-Popov ghosts, auxiliary field, and antifields, the divergent part of effective action possesses general covariance, and this important property holds in all orders of the perturbative loop expansion. This statement holds in all covariant models of QG, including the ones with higher derivatives and even for the nonlocal models. Starting from covariance and using power counting and locality of the counterterms, one can easily classify the models of QG into non-renormalizable, renormalizable, and superrenormalizable versions. One of the extensions of the analysis performed above is an extension to the nonlinear gauges [92], but this part is beyond the scope of the present review.

关闭量子度规、法捷耶夫-波波夫鬼场、辅助场和反场的平均场后，有效作用量的发散部分具备广义协变性，这一重要性质在微扰圈展开的所有阶中均成立。该结论适用于所有协变量子引力 (QG) 模型，包括高阶导数模型乃至非局域模型。从协变性出发，结合幂次计数与 counterterms 的局域性，即可轻松将量子引力模型分为不可重整化、可重整化和超可重整化三类。上述分析的一个拓展方向是非线性规范 [92]，但这部分内容超出了本综述的讨论范围。

Cross-References

交叉引用

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微扰量子引力背景信息

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